A graph-theoretic approach to explicit nonlinear pipe network optimization

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An explicit algorithm for nonlinear constrained pipe network optimization is developed. The explicit approach can be effectively used for directly determining a variety of pipe network characteristics to exactly satisfy defined values of quasi-linear boundary equality constraints. The constraint set represents stated supply pressure and volumetric flow requirements at designated critical nodes and pipes throughout the pipeline system for a range of operating conditions. The solution of the problem is based on an analytical reformulation of the quasi-linear steady-state network equilibrium equation set and the corresponding boundary specifications in terms of selected pipe system parameters. Owing to the presence of nonlinearity in these equations, the incremental Newton-Raphson method is utilized as the basic solution procedure. The solution, which is defined in a continuous variable space, is optimal in the sense that the decision parameters are calculated to meet the specified pressure and flow constraints. Every type of pipe conveying system can be optimized with this method. The solution space is secured through a well-arranged interaction between network topology, boundary constraints, and decision parameters. In order to illustrate the developed algorithm an example application is presented.

Keywords: pipe networks, nonlinear and explicit optimization, graph theory

Introduction

Increased research efforts have been directed toward developing algorithms for distribution network optimization. The various algorithms developed have taken the form of nonlinear, dynamic, heuristic, and successive linear programming economic models, which typically will attempt to size such network components as pipes, pumps, and elevated storage facilities based on component cost information. It is assumed that the basic network topology along with the location of the basic network components are fixed. In general, these algorithms are oriented toward optimal pipe size selection.[1] A partial review of the various approaches was previously provided.[1–4]

Despite the potential of such procedures their applications have been limited mainly owing to the complexity of the techniques implemented to yield the optimum solutions.[5] Our method incorporates some of the ideas proposed by Shamir and Howard[6] and Gofman and Rodeh[7]; however, the advantage it possesses relies on the utilization of the full set of continuity and energy relations. This full-equation approach allows the direct inclusion of both nodal pressure and pipe flow in the equality constraint set and has been shown to exhibit superior convergence characteristics.[8]

The proposed algorithm is based on recasting the network equation and constraint sets in terms of designated pipe system decision parameters. This results in an explicit solution for the decision parameters in a continuous variable space, i.e., theoretical values as opposed to discrete values. Parameters that can be explicitly determined include (1) design parameters such as pipe diameter, pipe length, pump power, pump head, storage level, and valve characteristics; (2) operating parameters such as pump speed, pressure-regulating valve setting, control valve setting, and flow or pressure specification; and (3) calibration parameters such as pipe roughness and node demands. Limited application of this approach to network design (pipe diameter optimization) and calibration (pipe roughness optimization) has been previously reported.[5,9]

The method can be readily applied to looped, branched, or combination networks of any size in which

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the fluid flow is assumed to be incompressible and isothermal. It is presented in an analytical form as a boundary value problem and results in an explicit optimum solution that does not require any sophisticated optimization packages. The solution is optimal in the sense that it exactly satisfies the operating specifications. The developed method is illustrated through an example application.

Topological properties of pipe networks

A pipe distribution network may be viewed as a directed linear graph and is composed of a collection of a finite number of edges (pipe sections, each with a specified length, diameter, and roughness) interconnected in a specified configuration. Pipe sections can contain pumps (or any other component whose differential head versus flow characteristic is known) and fittings, such as bends and valves. The endpoints of each edge are identified as either junction or fixed grade nodes.

Definition 1: Junction node. A junction node is a point where two or more edges join. It can also be a point of consumption where flow can enter and leave the network system.

Definition 2: Fixed grade node. A fixed grade node is a point where a constant energy grade is maintained, such as a connection to a reservoir, an elevated storage facility, or a constant pressure region.

Definition 3: Spanning tree. A spanning tree is a connected system of edges that contains all the nodes but does not contain any cycles. A system is said to be connected if it contains a path between any pair of its nodes, where a path is a sequence of edges that allows the movement from one node to another. A spanning tree must be rooted at any fixed grade node, which is then referred to as the datum node for the system.

All the edges that make up the spanning tree are referred to as branches, while the nontree edges are referred to as links. The nontree edges constitute the elements of the complement of the spanning tree, which is referred to as a cotree. Each link in the distribution network will uniquely form a closed path with its coterminous branches. Such a closed path, which is called a fundamental circuit, is independent of any other closed path, since it contains an edge (link) that no other fundamental circuit possesses. Since each link defines a unique fundamental circuit, it follows that in any connected system the Euler relation must hold:

$$e = n + l + s - 1 \tag{1}$$

Where e represents the total number of edges, n is the number of junction nodes, the cyclomatic number l is the number of fundamental circuits, and s is the number of fixed grade nodes. In addition, pseudo-circuits can be identified.

Definition 4: Pseudo-circuit. A pseudo-circuit is a chain of connected tree branches between the datum node and any other fixed grade node. It follows that in any connected network with s fixed grade nodes, the number of pseudo-circuits, p, equals $s - 1$.

Definition 5: Incidence matrix. Let $G' = (V,E)$ be a connected directed graph with v nodes and e edges. The incidence matrix $[A]$ of G' is a v by e matrix that has a row for every node and a column for every edge. For each column the nonzero row entries $+1$ and -1 indicate the nodes that begin and end an edge, respectively.

Definition 6: Fundamental circuit matrix. The fundamental circuit matrix $[E]$ for G' is an $e - v + 1$ by e matrix that has a row for each fundamental circuit in G' and a column for every edge. In each row the nonzero column entries of $+1$ or -1 indicate the presence of the directed edges in the circuit that go with or against the circuit orientation, respectively.

Observation 1. The rank of $[A]$ is $v - 1$. Furthermore, if any row of $[A]$ is omitted (assuming that the removal of the corresponding node does not disconnect the graph), the resulting matrix $[M]$ has a rank of $v - 1$ □.

Observation 2. The rank of $[E]$ is $e - v + 1$ □. The proofs for Observations 1 and 2 can be found in Ref. 10.

Definition 7: Junction node connectivity matrix. Let $G = (N,E)$ be a connected direct graph with $n = |N|$ junction nodes and $e = |E|$ edges. The junction node connectivity matrix $[\Lambda]$ of G is an n by e matrix that has a row for every junction node and a column for every edge. For each column the nonzero entries $+1$ and -1 indicate the junction nodes that begin and end an edge, respectively.

Definition 8: The *fp*-circuit basis matrix. The *fp*-circuit basis matrix $[\Gamma]$ of G is an $e - n$ by e matrix that has a row for each fundamental and pseudo-circuit in G and a column for every edge. In each row the nonzero entries, $+1$ or -1, indicate the presence of directed edges in the circuit that go with or against the circuit orientation, respectively.

Lemma 1. The rank of $[\Lambda]$ is n □.

Lemma 2. The rank of $[\Gamma]$ is $e - n$ □.

Lemma 3. The row vectors of $[\Lambda]$ and $[\Gamma]$ are mutually orthogonal, that is, $[\Lambda][\Gamma]^T = [0]$.

Proof. We need to show that for all i,j the inner product $\langle \lambda_i, \gamma_j \rangle = 0$, where λ_i denotes the ith row of $[\Lambda]$ and

γ_j denotes the jth row of $[\Gamma]$. Two arrangements are present:

1. The ith junction node is not connected to any of the edges in the jth circuit; then $\langle \lambda_i, \gamma_j \rangle = 0$.
2. The ith junction node is connected to exactly two edges in the jth circuit, implying presence of two nonzero entries in each of the corresponding column positions of λ_i and γ_j, respectively. But by construction of $[\Lambda]$ and $[\Gamma]$ the product of these four entries must be negative; hence $\langle \lambda_i, \gamma_j \rangle = 0$. This concludes the proof □.

Theorem 1. Let $[R]$ be a k by r matrix of rank k ($k < r$) and $[S]$ be an $r - k$ by r matrix of rank $r - k$. If $[R][S]^T = [0]$, then $[T] = \begin{bmatrix} [R] \\ [S] \end{bmatrix}$ is an r by r matrix of rank r.

Proof. We want to show that $[T]^T\{x\} = \{0\} \rightarrow \{x\} = \{0\}$. Let $\{x\} = \begin{Bmatrix} \{u\} \\ \{v\} \end{Bmatrix}$, where $\{u\}$ and $\{v\}$ are k by 1 and $r - k$ by 1 vectors, respectively. We now have

$$\{0\} = [T]^T\{x\} = [[R]^T[S]^T] \begin{Bmatrix} \{u\} \\ \{v\} \end{Bmatrix}$$
$$= [R]^T\{u\} + [S]^T\{v\}$$

It follows that

$$\{0\} = [R][R]^T\{u\} + [R][S]^T\{v\}$$
$$= [R][R]^T\{u\}$$

But the rank of $[R][R]^T$ is k; therefore $\{u\} = \{0\}$. Similarly, $\{0\} = [S]^T\{v\}$ implies $\{v\} = \{0\}$. So $\{x\} = \{0\}$. This concludes the proof □.

Corollary 1. The e by e matrix $\begin{bmatrix} [\Lambda] \\ [\Gamma] \end{bmatrix}$ is nonsingular.

Definition 9: Junction node to datum path matrix. The junction node to datum path matrix $[\Phi]$ of G is an n by e matrix that has a row for every junction node and a column for every edge. In each row the nonzero entries $+1$ or -1 indicate the presence of directed tree branches in the path connecting the junction node back to the datum node that go with or against the path orientation, respectively.

Observation 3. The rank of $[\Phi]$ is n □. The proof for this observation is given in Ref. 11.

Steady-state equilibrium relations

Up to this point, only the purely topological relations that characterize the pipe network problem have been examined. Regardless of the topological configuration of the distribution network, the flow of a fluid in the system is governed by two conservation laws.

Conservation of mass. Let $\{Q\}$ designate the column vector of volumetric flow rate associated with all the edges in the piping system; then the product of each row of $[\Lambda]$ with the flow vector must equal the column vector $\{q\}$ of external demands at the junction nodes (positive if inflow, negative if outflow). That is,

$$[\Lambda]\{Q\} = \{q\} \tag{2}$$

Conservation of energy. Let $\{\Psi\}$ designate the column vector of head loss associated with all the edges in the pipe system; then the product of each row of $[\Gamma]$ with the head loss vector must equal the column vector $\{K\}$ of differences in total grade between the two boundary circuit nodes (zero for fundamental circuits). That is,

$$[\Gamma]\{\Psi\} = \{K\} \tag{3}$$

The head loss variable, Ψ, associated with an edge includes the contribution of the frictional effects at the pipe wall boundaries, fittings and other local losses, and pumps (negative, i.e., head gain). This variable can be expressed as a nonlinear function of the edge flow as

$$\Psi = (\zeta - \alpha)Q^2 + \xi Q^\sigma - \beta\eta Q - \delta\eta^2 \tag{4}$$

in which ξ is a pipeline constant that is a function of the line length, diameter, and roughness; σ is an exponent that is dependent on the head loss equation used ($\sigma \epsilon [1.8,2]$); ζ is a pipeline constant that is a function of the sum of the minor loss coefficients for the fittings and the line diameter; and α, β, and δ are the coefficients of the pump characteristic curve associated with an edge that represents actual pump operation for a pump operating at a reference speed. For variable speed operation, η represents the ratio of the pump rotational speed at any time to the pump rotational speed associated with the data used to determine the coefficients[12] (reference speed). The pump characteristic curve describing the pump operation can be obtained from a pump test relating the discharge to the head differential across the pump.

Constructing the *fp*-circuit basis matrix $[\Gamma]$. The framing of the *fp*-circuit basis matrix $[\Gamma]$ requires that $l + p$ circuits be identified. These circuits can be generated using a breadth-first search (BFS) spanning tree building algorithm. A BFS spanning tree can be constructed recursively, utilizing only the connectivity pattern that describes the network topology as follows: Starting at a designated node j, all edges incident on j are traversed and identified as (j,j_1), (j,j_2), . . . , (j,j_m). The edges incident on $j_1, j_2, \ldots, j_m$ are then explored and identified as $(j_1,j_{1,1})$, $(j_1,j_{1,2})$, . . . , $(j_m,j_{m,km})$. The edges incident on $j_{1,1}, j_{1,2}, \ldots, j_{m,km}$ are then traversed, and so on. This process continues until all nodes have been visited. With this algorithm a two-step process is carried out. First, a BFS spanning tree is constructed starting at the datum fixed grade node and continuing until all the nodes in the system have been accessed. As each additional fixed grade node is accessed, the

chain of edges back to the datum fixed grade node is identified. This defines branches and links and p pseudo-circuits between fixed grade nodes. All edges connecting fixed grade nodes are then removed from further consideration. Second, a BFS spanning tree is generated starting at any nontree link and continuing until a tree branch, which is connected to the starting nontree link, is accessed. Only tree branches are utilized in this search. This continues until all nontree links have been utilized. At this time, l fundamental circuits will have been identified. This topological problem may also be solved using a Depth-First Search tree building algorithm as shown in Ref. 7.

Solution of the network flow problem

Equation (1) and Corollary 1 represent sufficient and necessary conditions for the solvability of the network flow problem, respectively. The first condition provides a means for checking the proper configuration of the distribution network and ensures the assembly of as many equations as there are unknowns. The second condition secures the framing of an independent set of flow equations. Since the resulting equation set is quasi-linear, the incremental Newton Raphson[6,13–15] can be utilized as the basic solution procedure. The preordering of these equations as measured by their degree of nonlinearity is unnecessary, since these equations are treated simultaneously by this method. The simultaneous solution of this system will determine the flow in each edge, i.e., network flow distribution. The column vector $\{H\}$ of energy grade at the junction nodes can then be computed as follows:

$$\{H\} = H_D\{I\} + [\Phi]\{\Psi\} \tag{5}$$

where H_D is the energy grade at the datum node and $\{I\}$ is a unit column vector.

Explicit optimization algorithm

Problem formulation

The optimization of a distribution piping network consists of determining selected physical edge-component characteristics to satisfy desired system performance conditions. These conditions are normally expressed as edge flow and junction node pressure (or grade) requirements, which are allowed in the system for a range of operating conditions. The optimal solution could be obtained by a repetitive trial-and-evaluation procedure of the network flow problem, until the pressure and flow requirements are eventually satisfied. The trial-and-evaluation procedure will generally result in a less than optimum solution and may be effective when only one or two edge-component characteristics and corresponding pressure and flow requirements are considered. However, as the number of parameters and conditions increases, the trial-and-evaluation procedure may not be effective or reliable and in fact may not even work. The result of utilizing this approach will often be inefficient performance at greater cost.

Our approach provides a direct, rapid calculation of the physical parameters that exactly meet the stated specifications. A continuous variable space is assumed for the solutions. The objective is to simultaneously satisfy the network conservation laws and the prescribed system boundary equality constraints. The full-equation approach presented earlier is utilized for this purpose. Since the assembled flow equations are functions of length, diameter, roughness, minor loss coefficient, pump speed ratio, and other pipe system characteristics, it is possible to explicitly solve for additional parameters other than the basic unknown variables (i.e., the pipe flow rates) to meet the equality conditions imposed. This can be done by introducing a new equation for each additional indeterminate parameter to the full equation set. The augmented set of variables represents the decision variables for the explicit optimization algorithm. An energy equation is added when the energy grade or pressure at a particular junction node is specified. Such a junction node is then referred to as a critical node. This equation is written as a pseudo-circuit for a pipe path between the critical node and the datum node for the system, which incorporates the added variable. A continuity equation is added when the velocity or volumetric flow condition into a particular pipe is specified. Such a pipe is then referred to as a critical pipe. This equation is written for a fictitious junction node of "degree 1" connecting the critical pipe. That is,

$$Q_i = \rho_i \tag{6}$$

in which ρ_i designates the specified volumetric flow rate in critical pipe i. The degree of a node is defined as the number of edges incident to that node. Additional boundary conditions, and thus equations, may be added if so desired. Each added specification (expressed as stated pressure or flow equality constraint) will allow the explicit determination of an additional edge-component parameter. The augmented system of equations is then recast analytically in terms of pipe flow rates and indeterminate pipe system parameters.

Global multiplication factor

Instead of determining a single edge-component parameter per additional continuity or energy equation, a global multiplication factor may be used for the calculation of multiple indeterminate parameters. This factor, which will multiply decision variables in all or a group of selected edges, can be computed in order to satisfy the equality condition imposed. Again, any conditions (and thus equations) may be added, if needed. This will result in one additional global multiplication factor for each additional equation. When more than one boundary condition is specified, the pipeline system may be partitioned into several regions. The number of regions must be equal to the number of specified conditions, and a different global factor for each region is calculated. This factor is then used to multiply all decision variables included in its respective region.

Each region will consist of a set of pipes with one unknown edge-component characteristic, which may differ from one region to the other. In addition, some pipes may be excluded from these regions and would thus be kept unaffected.

Basic solution procedure

The explicit optimization problem is reduced to solving a system of algebraic equations,

$$\{F(\{Q\},\{\theta\})\} = \{0\}$$
$$f_i = 0,\ i = 1, 2, \ldots, e + d \quad (7)$$

in which $\{\theta\}$ designates the column vector of specified decision variables and e and d are the dimensions of vectors $\{Q\}$ and $\{\theta\}$, respectively. Since these equations are quasi-linear, an iterative procedure is required to obtain the final solution. The method used here is the incremental Newton Raphson (NR), where the linearized set of equations,

$$[F'(\{Q\},\{\theta\})]^{(\nu)} \begin{Bmatrix} \{\Delta Q\} \\ \{\Delta\theta\} \end{Bmatrix}^{(\nu+1)} = -\{F(\{Q\},\{\theta\})\}^{(\nu)}$$
$$\nu \geq 0 \quad (8)$$

is solved iteratively. Here, $\{\Delta Q\}$ signifies the incremental edge flow column vector, and $\{\Delta\theta\}$ signifies the column vector of incremental decision variables. The superscript in parentheses designates the Newton's iteration number, with ν being the current Newton's step. The Jacobian matrix $[F']$ is given by

$$[F'(\{Q\},\{\theta\})] = \begin{bmatrix} \frac{\partial f_1}{\partial Q_1} \cdots \frac{\partial f_1}{\partial Q_e} \frac{\partial f_1}{\partial \theta_1} \cdots \frac{\partial f_1}{\partial \theta_d} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{\partial f_{e+d}}{\partial Q_1} \cdots \frac{\partial f_{e+d}}{\partial Q_e} \frac{\partial f_{e+d}}{\partial \theta_1} \cdots \frac{\partial f_{e+d}}{\partial \theta_d} \end{bmatrix}, \quad (9)$$

with its coefficients being determined explicitly. Closed form expressions for the coefficients can be easily derived as shown in Ref. 15.

At each iteration, improved values for $\{Q\}$ and $\{\theta\}$ are obtained from

$$\{Q\}^{(\nu+1)} = \{Q\}^{(\nu)} + \{\Delta Q\}^{(\nu+1)} \quad (10a)$$
$$\{\theta\}^{(\nu+1)} = \{\theta\}^{(\nu)} + \{\Delta\theta\}^{(\nu+1)} \quad (10b)$$

The iterations continue until both sets of flow rates and decision variables converge to consistent values (i.e., a solution is achieved).

Illustrative example

The proposed algorithm is best illustrated by using the simple network example shown in *Figure 1*. This network comprises five directed edges (e_1, e_2, e_3, e_4, e_5), three junction nodes (n_1, n_2, n_3), and two fixed grade nodes (s_1, s_2). Fixed grade node s_1 is identified as the datum node, and a possible spanning tree will consist of edges (branches) e_1, e_2, e_3, and e_5. The pseudo-circuit then consists of branches e_1, e_3, and e_5. The

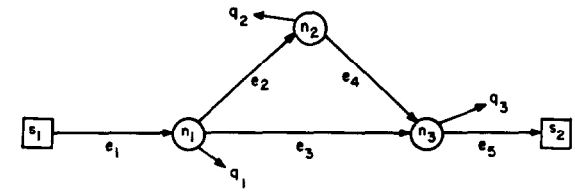

Figure 1. Example of network

nontree edge (link), i.e., e_4, defines a fundamental circuit, i.e., (e_2, e_3, e_4). Junction node n_2 is designated as a critical node with a specified energy grade H_{n_2}, and θ is designated as a decision variable for edge e_1. This defines an additional pseudo-circuit, i.e., (e_1, e_2). The mathematical model can be formulated as follows:

Continuity

$$\begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 1 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{Bmatrix} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \quad (11)$$

Energy

$$\begin{bmatrix} 0 & -1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Psi_1(Q_1,\theta) \\ \Psi_2(Q_2) \\ \Psi_3(Q_3) \\ \Psi_4(Q_4) \\ \Psi_5(Q_5) \end{Bmatrix} = \begin{Bmatrix} 0 \\ H_{s_1} - H_{s_2} \\ H_{s_1} - H_{n_2} \end{Bmatrix} \quad (12)$$

in which H_{s_1} and H_{s_2} are the energy grades of s_1 and s_2, respectively. The solution of this quasi-linear system by the NR method will give the flow rate vector $\{Q\}$ and the decision variable θ.

Application

The developed algorithm has widespread applications associated with the design, operation, calibration, future planning, evaluation, expansion, real-time modelling, and improvement of fluid distribution networks. It allows for a variety of decision parameters to be explicitly determined while meeting required system performance criteria. Pipeline characteristics, pump characteristics, fluid level in storage facilities, other network element characteristics, or any of their combinations can be selected as decision variables and can be directly included in the solution while satisfying explicit boundary equality constraints. These represent required pressures and flow rates to be maintained at specified locations in the system. Such capabilities will allow practicing engineers to provide sound decision making and to conceive and evaluate efficient and reliable alternatives or recommendations with reference to suggested or required system performance.

In order to demonstrate the feasibility, flexibility, and wide range of capabilities of the developed method it was applied to an example pipeline network. The sample network is illustrated in *Figure 2*. This network can be considered as a simplified but reasonably typical municipal water distribution system. It comprises 19 pipe sections, 13 junction nodes, four fundamental circuits, and three fixed grade nodes, i.e., two pseudo-circuits. The fundamental circuits consist of (1) pipes 2, 5, 6, and 7; (2) pipes 7, 8, 9, 10, and 16; (3) pipes 3, 10, 11, 12, and 14; and (4) pipes 3, 10, 14, 15, and 16. Fixed grade node A is identified as the datum node for the system, and the pseudo-circuits consist of (1) pipes 1, 2, 3, and 4 and (2) pipes 1, 2, 10, 11, and 13. SI units and the Hazen-Williams head loss expression were utilized for this example. The pipeline constants are given by

$$\zeta = 0.08265 \frac{K}{D^4} \tag{13}$$

$$\xi = \frac{10.69\, L}{C^{\sigma} D^{4.87}} \tag{14}$$

and the exponent $\sigma = 1.852$. Here, K signifies the sum of the minor loss coefficients, D is the pipe diameter in meters, and L is the pipe length in meters. *Table 1* summarizes the pertinent pipe system characteristics. The assumed flow direction is from the first (head) to second (tail) node input. Water level in storage facilities and external demands at junction nodes throughout the distribution network are indicated in *Figure 2*, while the pump characteristic curve is depicted in *Figure 3*. The explicit algorithm was utilized in optimizing this system for four different cases. In all cases the optimum solution was obtained after two or three Newton iterations. The convergence tolerance was set to 0.001.

Case 1

The objective of this application was to verify that our approach is capable of reproducing known situations. A hydraulic analysis ($d = 0$), which requires the simultaneous solution of 13 continuity equations and six energy equations, was carried out for the set of operating conditions described earlier. The resulting

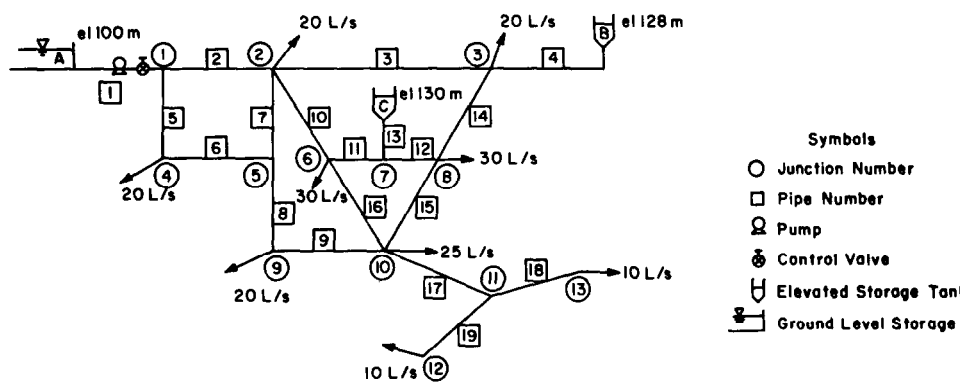

Figure 2. Sample pipe distribution network

Table 1. Pipe system characteristics

Pipe number	Head node no.	Tail node no.	Length (m)	Diameter (mm)	Roughness coefficient	Minor loss coefficient	Node number	Demand (l/sec)
1	A	1	300.0	300.0	120.0	0.0	1	0.0
2	1	2	250.0	250.0	120.0	0.0	2	20.0
3	2	3	450.0	250.0	120.0	0.0	3	20.0
4	3	B	300.0	200.0	120.0	0.0	4	20.0
5	1	4	150.0	250.0	120.0	0.0	5	0.0
6	4	5	250.0	200.0	120.0	0.0	6	30.0
7	5	2	150.0	200.0	120.0	0.0	7	0.0
8	5	9	170.0	200.0	120.0	0.0	8	30.0
9	9	10	200.0	250.0	120.0	0.0	9	20.0
10	2	6	160.0	200.0	120.0	0.0	10	25.0
11	6	7	140.0	200.0	120.0	0.0	11	0.0
12	8	7	80.0	250.0	120.0	0.0	12	10.0
13	7	C	200.0	250.0	120.0	0.0	13	10.0
14	3	8	200.0	250.0	120.0	0.0		
15	10	8	300.0	200.0	120.0	0.0		
16	6	10	300.0	200.0	120.0	0.0		
17	10	11	300.0	150.0	120.0	0.0		
18	11	13	200.0	150.0	120.0	0.0		
19	11	12	175.0	150.0	120.0	0.0		

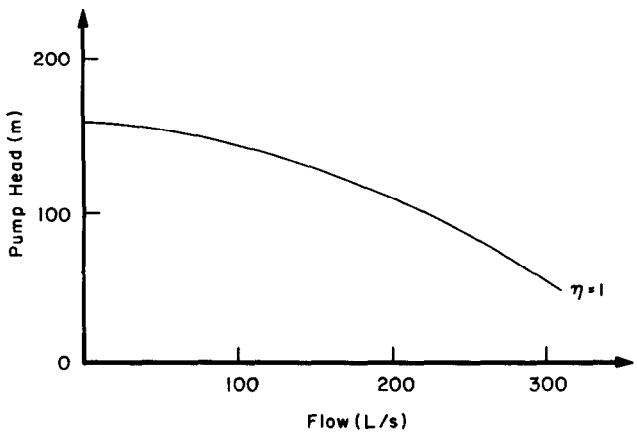

Figure 3. Pump characteristic curve

Table 2. Application results: Pipe flow rates

Pipe number	Flow (l/sec)				
	Original	Case 1	Case 2	Case 3	Case 4
1	281.99	281.99	329.76	221.12	249.44
2	177.07	177.07	208.98	136.48	129.74
3	89.10	89.10	109.23	64.55	83.06
4	43.62	43.62	56.27	32.71	37.51
5	104.92	104.92	120.78	84.64	119.70
6	84.92	84.92	100.78	64.64	99.70
7	12.07	12.07	15.62	8.10	22.25
8	72.85	72.85	85.16	56.54	77.44
9	52.85	52.85	65.16	36.54	57.44
10	80.04	80.04	95.37	60.03	48.94
11	37.59	37.59	53.95	17.98	19.03
12	15.78	15.78	35.54	−14.57	7.90
13	53.37	53.37	88.49	3.41	26.93
14	25.47	25.47	32.97	11.84	25.55
15	20.30	20.30	31.58	3.59	12.35
16	12.45	12.45	11.42	12.05	−0.09
17	20.00	20.00	20.00	20.00	20.00
18	10.00	10.00	10.00	10.00	10.00
19	10.00	10.00	10.00	10.00	10.00

pipe flow rates and junction node grades are presented in *Tables 2* and *3*. These results provide a sound basis for evaluating the applicability and capability of the proposed algorithm to reproduce known situations. This can be achieved by altering the original system parameters, such that certain adjustments will be required to return the modified system to its original status. The altered network parameters are selected as decision variables, and the flow rates and pressures determined from the hydraulic analysis are used as boundary constraints in the explicit optimization algorithm. The results of the algorithm can then be compared with those of the original system. Obviously, the results should be identical. The selected decision variables consist of the pump speed ratio for the pump in pipe 1 and the elevation of tank B. The selected boundary constraints consist of a specified flow of 43.62 l/sec in pipe 4 and a grade of 133.14 m at node 9. The calculated reservoir elevation and pump speed ratio are 128.0 m and 1.0, respectively, which are effectively identical to their

Table 3. Application results: Junction node grades

Junction number	Grade (m)				
	Original	Case 1	Case 2	Case 3	Case 4
1	151.11	151.11	160.91	141.72	154.81
2	138.06	138.06	143.17	133.66	136.41
3	131.47	131.47	133.56	130.04	130.62
4	148.14	148.14	157.05	139.73	151.02
5	138.22	138.22	143.42	133.74	137.66
6	132.36	132.36	135.29	130.32	130.67
7	131.13	131.13	132.89	130.01	130.32
8	131.18	131.18	133.09	129.97	130.33
9	133.14	133.14	136.64	130.56	131.97
10	132.02	132.02	135.00	130.00	130.67
11	128.70	128.70	131.68	126.68	127.35
12	128.16	128.16	131.14	126.14	126.00
13	128.09	128.09	131.06	126.06	125.81

original values. The computed flow rates and junction node grades are shown in *Tables 2* and *3*.

Case 2

In the second application the explicit optimization algorithm was used to compute the pump speed ratio for the pump in pipe 1 to meet a hydraulic grade of 135 m at junction node 10. The calculated pump speed ratio is 1.1426. The resulting pipe flow rates and junction grades are presented in *Tables 2* and *3*.

Case 3

For this application the minor loss coefficient for the control valve in pipe 1 was calculated to satisfy a hydraulic grade equality constraint of 130 m at node 10. The solution of the modified equation set (13 continuity and seven energy equations) gives a value of 97.41 for the control valve coefficient. The resulting pipe flows and nodal grades are depicted in *Tables 2* and *3*. Negative signs for flow rates denote that the flow is opposite to the assumed direction.

Case 4

For the final application the roughness coefficient for pipes 1, 2, 7, 10, 16, 18, and 19 were adjusted to meet a specified grade of 126 m at node 12. The decision variable consists of a global roughness multiplication factor, which adjusts the roughness coefficient of the selected set of pipes. The solution of the modified equation set (13 continuity and seven energy equations) gives a value of 73.1 for the roughness of the selected pipes. The calculated flow rates and junction node grades are displayed in *Tables 2* and *3*.

Conclusions

The technique described in this work for nonlinear constrained pipe network optimization has been applied with considerable success. Every type of distribution system can be analyzed and optimized to exactly satisfy defined values for boundary nodes and pipe sections for a range of operating conditions. For a distribution network with a fixed flow pattern, such

as a tree network with only one fixed grade node or source of supply, the boundary conditions are limited to pressure equality constraints to be met at particular critical junction nodes. A variety of system parameters and any of their combinations can be explicitly determined. Such capabilities will greatly enhance the ability of engineers to effectively utilize hydraulic network modelling to efficiently design, operate, and calibrate pipe distribution systems.

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