Computer modeling of water quality in large multiple-source networks

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A simple and efficient computer-oriented methodology is presented for use in monitoring and modeling of steady-state water quality patterns in large multiple-source networks. The method is formulated analytically as a boundary value problem from mass balance relationships. It produces an explicit solution for the supply source contribution to delivered flows, chemical concentrations, and a range of water age parameters at designated junction nodes throughout the distribution system for a specified set of operating and loading conditions. The method is robust and reliable, and it is guaranteed to converge in an expeditious manner. The wide range of capabilities and utility of the method are illustrated by using an example network.

Keywords: pipe networks, flow distribution, water quality

Introduction

In the field of water supply management, one is often confronted with the problem of assessing the quality of delivered water subject to hydraulic mixing effects in multiple-supply source networks. These assessments require a comprehensive water quality modeling capability that includes an accurate calculation of network flow hydraulics and various water quality distributed parameters for a given set of network operational and loading conditions. The parameters include spatial distribution of concentrations and the associated water age distribution throughout the system.

In recent years, various mathematical models have been developed for use in predicting the effects of hydraulic behaviors on the variability of water quality in networks. These include steady-state, sequential steady-state, and dynamic water quality models. A comprehensive review of the various approaches was previously provided.[1–3] However, there still remains a need for a more efficient and direct approach.

This paper presents a computer-oriented methodology for use in monitoring and modeling steady-state water quality patterns in large multiple-source networks. The method is formulated analytically from mass balance relationships as a sequence of contingent boundary value problems. It produces an explicit and rapid solution for the supply source contribution to delivered flows, chemical concentrations, and a range of water age parameters at critical locations throughout the system for a specified set of operating and loading conditions. The method is based on some of the ideas proposed by Males et al.,[4] Clark et al.,[5] and Wood and Ormsbee[6]; however, the advantage it possesses relies on the formulation and direct solution of the contingent boundary value problems. Such an approach eliminates the need for hydraulic ordering of nodes and edges and allows parameter matrix formulations to be

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Received 3 October 1991; revised 30 January 1992; accepted 14 February 1992

explicitly solved with a reduced computer storage requirement. A graph-theoretical procedure is utilized to determine the maximum and minimum routing times of water through the network. Circulating flows, singularity problems, and remedial measures are also addressed. The wide range of capabilities and utility of the method are illustrated by using an example network. Before the method is examined in detail, a short summary of the associated network equations is provided.

Network definition

A water-conveying system can be represented by a directed linear graph consisting of a finite number of interconnected elements. A graph element consists of an oriented edge (pipe segment) and its two distinct terminal nodes. Each edge has a defined length, diameter, roughness, and material. Edges may contain pumps (or any other hydraulic component whose head flow rate functional characteristic is known) and fittings, such as bends, meters, and valves, where concentrated energy dissipation occurs. Each terminal node is identified as either a junction or a fixed grade node. A junction node is a point (of known elevation) where two or more edges join. It can also be a point of external consumption where flow can be injected or removed from the graph. A source node is a point of constant energy grade such as a connection to a well, a treatment plant and its pumping station, a constant pressure region, or a discharging tank or reservoir. Edges and nodes are uniquely identified by labels. Their characteristics and incidence patterns are stored in a data base. This allows for a rapid and efficient data manipulation for subsequent deletion, addition, or modification of any element in the network. The edge-node interconnection uniquely defines the network topology with all independent circuits being identified. Each circuit is defined as either a fundamental circuit or a pseudo-circuit. A fundamental circuit is a closed sequence of connected edges that uniquely contains exactly one edge that no other circuit possesses. A pseudo-circuit is a sequence of connected edges between the root and any other source node. Algorithms for the construction of these circuits have been previously described by Osiadacz,[7] Boulos and Altman,[8] and Boulos and Ormsbee.[9] For a connected graph containing e edges, n junction nodes, and s source nodes the number of pseudo-circuits is $s - 1$, while the number of fundamental circuits l is defined by the Euler equation,

$$l = e - n - s + 1 \tag{1}$$

Network flow problem

The flow in water networks satisfies two physical laws. The continuity law implies that for each junction node the algebraic sum of flows is zero. The mechanical energy balance law implies that along each circuit (fundamental as well as pseudo) the algebraic sum of energy loss (including minor loss) minus any energy introduced by pumps is zero. The analytical flow problem can be formulated as

$$q_j + \sum_{i=1}^{e} \lambda_{j,i} Q_i = 0 \qquad j = 1, \ldots, n \tag{2a}$$

$$\Delta\Phi_m + \sum_{i=1}^{e} \gamma_{m,i}\left[\xi_i Q_i^{\sigma} + \zeta_i Q_i^2 - \eta^2\left(\alpha - \frac{\beta}{\eta^{\nu}} Q_i^{\nu}\right)\right] = 0 \qquad m = 1, \ldots, l + s - 1 \tag{2b}$$

or, in a more compact form,

$$\{F(\{Q\})\} = \{0\} \qquad f_i = 0 \quad \text{for} \quad i = 1, \ldots, e \tag{3}$$

in which i and j are the edge and junction labels, respectively; m denotes the circuit label; q is the injected flow rate (negative, if outflow); Q is the volumetric edge flow rate; $\lambda_{j,i}$ is 0 when edge i is not incident to junction node j or 1, -1 when edge i is directed toward or away from node junction j; $\Delta\Phi_m$ is zero for fundamental circuits or it denotes the difference in energy grade between the two boundary source nodes in circuit m; $\gamma_{m,i}$ is zero if edge i does not belong to circuit m, otherwise it is set to 1 or -1 depending on whether edge i belongs to circuit m with the same or opposite orientation; ξ_i represents the resistance in edge i that is a function of edge length, diameter, and material; σ is an exponent that depends on the energy loss expression used, $\sigma \in [1.8, 2.0]$; ζ_i is the minor loss resistance that is a function of the sum of the minor loss coefficients for the fittings of edge i and of its diameter; α is the pump cutoff head at zero flow condition; β and ν are the regression coefficient and the exponent of the pump characteristic curve that represents actual pump operation for a pump operating at a reference speed; and η is the ratio of the pump rotational speed to the pump reference speed. The pump operation can also be described by a quadratic characteristic curve.[8,10,11]

The solution of this quasi-linear boundary value problem will determine the volumetric flow rate in each edge. For this, an iterative procedure, such as the Newton-Raphson method, is expressed as follows:

$$[K(\{Q\})]^{(i)}\{\Delta Q\}^{(i+1)} = -\{F(\{Q\})\}^{(i)} \tag{4a}$$

$$\{Q\}^{(i+1)} = \{Q\}^{(i)} + \{\Delta Q\}^{(i+1)} \tag{4b}$$

in which $\{\Delta Q\}$ is the incremental flow vector. The tangent or Jacobian matrix K, which is defined as the first partial differential matrix of $\{F\}$ with respect to $\{Q\}$, is given by

$$[K(\{Q\})] = \begin{bmatrix} \frac{\partial f_1}{\partial Q_1} & \cdots & \frac{\partial f_1}{\partial Q_e} \\ \vdots & & \vdots \\ \frac{\partial f_e}{\partial Q_1} & \cdots & \frac{\partial f_e}{\partial Q_e} \end{bmatrix} = [k_{i,j}]$$

$$i,j = 1, \ldots, e \quad (5)$$

with its coefficients being explicitly determined at each iteration from

$$k_{i,j} = \begin{cases} \lambda_{i,j} \\ \gamma_{i,j}[\sigma\xi_j Q_j^{(\sigma-1)} + 2\zeta_j Q_j + \nu\beta\eta^{(2-\nu)}Q_j^{(\nu-1)}] \end{cases} \quad (6)$$

The iterative process continues until the relative change in the flow rates is less than some specified tolerance. Such an approach has proved to be robust, and it exhibits excellent convergence characteristics.[12,13]

The resulting coefficient matrix is sparse; i.e., it has relatively few nonzero elements. Furthermore, its sparsity pattern remains constant throughout. It follows that since the structure of the matrix is preserved, sparse matrix routines can be utilized even more efficiently. Two matrices are said to be of the same structure if they contain nonzero entries in the same positions.

Source to junction node influence problem

The ability to determine the influence characteristics of supply sources to nodal loads in water-conveying networks is necessary for proper design when supply sources with various pollutants or chemicals are present. The understanding and the prediction on potential water quality changes with varying source influences or supply sources going on-off line are also vital to ensure meeting the consumption requirements at acceptable quality. When multiple-supply sources are feeding the distribution network, the influence characteristic of each source to the consumption at each junction node can be explicitly determined from influence continuity based on previously calculated flow distribution and assuming complete mixing at junction nodes. The analytical source influence problem can be formulated as follows:

$$\left(\sum_{i=1}^{n} \delta_{j,i} Q_{j,i} - q_j\right) I_{k,j} - \sum_{i=1}^{n} \delta_{j,i} Q_{i,j} I_{k,i} = \delta_{k,j} Q_{k,j}$$

$$j = 1, \ldots, n \quad \text{and} \quad k = 1, \ldots, s \quad (7)$$

with

$$\sum_{j=1}^{n}\sum_{k=1}^{s} I_{k,j} = n \quad \text{if} \quad q_j \leq 0 \quad \text{for} \quad j = 1, \ldots, n \quad (8)$$

in which $\delta_{v,w}$ is 1 if node w is adjacent to node v and 0 otherwise; $Q_{v,w}$ is the volumetric flow rate from node v toward node w; and $I_{k,w}$ denotes the volumetric influence characteristic of supply source k to node w and represents the fraction of the outflow from junction node w emanating at supply source k. The resulting boundary value problem is thus represented by a sequence of n simultaneous linear algebraic equations with constant coefficient matrix, but with s successive right-hand sides. For each equation set the right-hand side is the force vector, which consists of the outflow from the designated supply source. It follows that the (sparse) coefficient matrix needs to be assembled and factorized only once. The s successive back substitution steps are then carried out to obtain the desired solution.

For a junction node where no flow emanates from the supply source, e.g., a dead-end node, an underdetermined system of nodal influence equations will result. This phenomenon is characterized by a row of zeros in I. Such a junction node satisfies the null diagonality condition,

$$\sum_{i=1}^{e} \delta_{j,i} Q_{j,i} = q_j \quad (9)$$

in which j is the junction node label. Calculations can still be carried out by setting the diagonal entry for the designated junction node to unity. The solution of the modified nodal influence equation set will yield a zero volumetric influence characteristic for that node.

For water quality calculations the total concentration of injected chemicals at each junction node emanating from the supply sources feeding the distribution network can be determined from

$$C_j = \sum_{k=1}^{s} I_{k,j} C'_k \qquad j = 1, \ldots, n \quad (10)$$

in which C_j is the total concentration of the chemicals at junction node j and C'_k is the known chemical concentration in boundary supply source k. The derivation of equation (10) is based on the assumption that the chemicals being considered are conservative, i.e., nonreactive and nondegradable, with complete mixing at the junction nodes.

Water age problem

Recently, considerable attention has been focused on the problem of determining the flow routing times from supply sources to critical junction nodes. The problem is of particular interest for water quality management, identification of inefficient flow paths, and the analysis of time-dependent chemical and waterborne pollutant characteristics. Water ages as it proceeds through the network, and this aging process is directly dependent on the spatial distribution of flows and the flow pattern taken.

For branched networks the flow pattern is uniquely defined, while for circuited networks it is not. In the latter case the water may be routed through more than one path from the supply source to a junction node. Since multiple flow patterns may be present in a distribution network, the age of the water may not be uniquely characterized, and hence the minimum, maximum, and average water age characteristics are used to describe the propagation process.

Determination of the minimum/maximum age of water

Let G be a connected directed graph with n junction nodes, s supply sources, and e edges with nonnegative flow times associated with the edges. The flow time through an edge is the ratio of the edge length to the edge flow velocity. The time length along a flow path between two nodes is the sum of the flow times of its edges, where a flow path is a sequence of edges that allows for the movement of flow from the source to the destination node. The minimum age of water arriving at a junction node from a designated supply source node is the shortest time length of the flow path between the two nodes. The set of single supply source to all junction nodes shortest time length flow paths can be obtained as shown below. Starting at the designated source node v of G, the graph is traversed level by level. The level of a junction node w is the length of the shortest path (in terms of number of edges) from v to w. The traversal proceeds as follows: All of v's children are visited in an arbitrary order. As each child w is accessed, the age of the arriving water is calculated and compared to its previously computed value with the minimum value (and the path corresponding to it) being retained. All reachable edges are traversed, some more than once, excluding edges with zero flow. The next nodes to be visited include all of v's "grandchildren" with their associated water ages calculated in the same fashion. Junction nodes at a new level may be visited if and only if all of the nodes at the previous level have already been visited. A minimum water age value of zero is assigned to all remaining unaccessed junction nodes. This process continues until no new levels can be reached. The above algorithm is repeated for each supply source as needed. Alternatively, one can apply the well-known Dijkstra's algorithm to solve the minimum water age problem.

The maximum age of water arriving at a junction node from a designated supply source is the longest time length of the flow path between the two nodes. The problem of finding the longest time length flow path between two nodes in a cyclic graph is known to be NP-complete and hence almost certainly intractable.[14] A directed graph is said to be cyclic if there exists at least one pair of nodes, say (v,w), such that there exists a path from v to w and from w to v. Otherwise the graph is said to be acyclic. This situation arises whenever a pump is positioned in an edge that belongs to a fundamental circuit. The energy conservation principle will dictate a flow circulation in the direction the pump is located, i.e., from the suction side to the discharge side of the pump. For the class of acyclic graphs a straightforward modification of retaining the maximum (as opposed to minimum) water age value and its associated path for each visited junction node allows for a solution through a direct application of the above shortest time length flow path algorithm.

Average age of water

This problem requires the determination of the flow-weighted average age from a designated supply source to every junction node. The network flow distribution is modified to account for the flow influence characteristics of the supply source node considered. The analytical flow-weighted average nodal age problem can be formulated as

$$\left(\delta_{j,k}Q_{k,j} + \sum_{i=1}^{n} \delta_{j,i}I_{k,i}Q_{i,j}\right)\theta_{k,j} - \sum_{i=1}^{n} \delta_{j,i}I_{k,i}Q_{i,j}\theta_{k,i} = \delta_{j,k}Q_{k,j}(\theta'_k + t_{k,j}) + \sum_{i=1}^{n} \delta_{j,i}I_{k,i}Q_{i,j}t_{i,j} \qquad j = 1,\ldots,n \quad \text{and} \quad k = 1,\ldots,s \quad (11)$$

in which $\theta_{k,w}$ is the average age of water at junction node w that originates from supply source k; θ'_k is the known value of average age of water at supply source k; and $t_{v,w}$ is the flow time through the edge connecting v and w. For each individual supply source the average age of water at every junction node is determined by solving a sparse system of n simultaneous linear algebraic equations. The flow-weighted average age of water at junction nodes associated with all supply sources feeding the distribution system can be easily determined by setting the influence characteristic values to unity. For a junction node that is not fed from the supply source node under analysis, the diagonal entry for that node is set to one.

Illustrative example

Figure 1 shows the schematic of a sample network studied earlier by Males et al.[4] for the source contribution to the nodal concentration problem. This network comprises seven directed edges $(e_1,e_2,e_3,e_4,e_5,e_6,e_7)$, four junction nodes (n_1,n_2,n_3,n_4), and two supply sources (s_1,s_2). The edge flow rates and

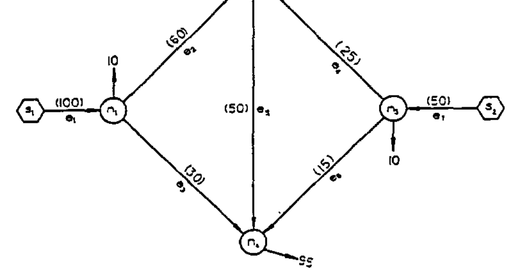

Figure 1. Example of network

nodal loads are also depicted in *Figure 1*. The source to junction nodes influence characteristics can be determined by solving equation (7) for each individual supply source. For the sample network the coefficient matrix is assembled as follows:

$$\begin{bmatrix} (Q_2+Q_3-q_1) & 0 & 0 & 0 \\ -Q_2 & (Q_5-q_2) & -Q_4 & 0 \\ 0 & 0 & (Q_4+Q_6-q_3) & 0 \\ -Q_3 & -Q_5 & -Q_6 & -q_4 \end{bmatrix} \tag{12}$$

with the force vectors being $(Q_1,0,0,0)^T$ and $(0,0,Q_7,0)^T$ for supply sources s_1 and s_2, respectively. On the basis of the flow distribution and nodal loads given in *Figure 1* the influence characteristic vectors of supply sources s_1 and s_2 to junction nodes are computed as $(1.0,0.7059,0.0,0.6873)^T$ and $(0.0,0.2941,1.0,0.3127)^T$, respectively. Assuming that the chemical concentration at supply source s_1 is C_1' and that the chemical concentration at supply source s_2 is C_2', the total concentration of the chemicals at the junction nodes can be determined from equation (10) as

$$C_1 = 1.0(C_1') + 0.0(C_2') = C_1' \tag{13a}$$

$$C_2 = 0.7059(C_1') + 0.2941(C_2') = 0.7059C_1' + 0.2941C_2' \tag{13b}$$

$$C_3 = 0.0(C_1') + 1.0(C_2') = C_2' \tag{13c}$$

$$C_4 = 0.6873(C_1') + 0.3127(C_2') = 0.6873C_1' + 0.3127C_2' \tag{13d}$$

These results are in agreement with the ones presented by Males et al.[4]

Application

In order to demonstrate the utility and capabilities of the present approach it was applied to an example distribution network shown in *Figure 2*. A labeling scheme is shown for edges, junction nodes, and supply sources. This network can be considered as a simplified but reasonably typical municipal water-conveying

Table 1. Edge characteristics

Edge number	Head node number	Tail node number	Length (m)	Diameter (mm)	Roughness coefficient	Flow rate (l/sec)	Velocity (m/sec)	Flow time (hours)
1	*A*	1	600.0	1000.0	120.0	1492.27	1.90	0.09
2	1	2	1300.0	800.0	120.0	335.36	0.67	0.54
3	3	2	400.0	800.0	120.0	245.36	0.49	0.23
4	4	3	300.0	800.0	120.0	721.13	1.43	0.06
5	*B*	4	400.0	1000.0	120.0	1507.73	1.92	0.06
6	1	5	400.0	800.0	120.0	956.91	1.90	0.06
7	5	6	500.0	800.0	120.0	299.84	0.60	0.23
8	7	6	800.0	800.0	120.0	401.35	0.80	0.28
9	2	7	400.0	800.0	120.0	480.72	0.96	0.12
10	8	7	400.0	800.0	120.0	462.37	0.92	0.12
11	3	8	400.0	800.0	120.0	275.77	0.55	0.20
12	9	8	300.0	800.0	120.0	386.60	0.77	0.11
13	4	9	400.0	800.0	120.0	586.60	1.17	0.10
14	5	10	400.0	800.0	120.0	557.07	1.11	0.10
15	10	11	500.0	800.0	120.0	357.07	0.71	0.20
16	6	11	400.0	800.0	120.0	601.19	1.20	0.09
17	11	12	800.0	800.0	120.0	233.67	0.47	0.48
18	7	12	400.0	600.0	120.0	441.73	1.56	0.07
19	11	13	300.0	600.0	120.0	624.60	2.21	0.04
20	13	14	300.0	600.0	120.0	183.44	0.65	0.13
21	15	14	500.0	600.0	120.0	162.71	0.57	0.24
22	12	15	300.0	600.0	120.0	575.40	2.04	0.04
23	13	16	300.0	600.0	120.0	241.16	0.85	0.10
24	17	16	300.0	600.0	120.0	8.49	0.03	2.78
25	14	17	300.0	600.0	120.0	146.15	0.52	0.16
26	18	17	500.0	600.0	120.0	112.69	0.40	0.35
27	15	18	300.0	600.0	120.0	212.69	0.75	0.11
28	16	19	200.0	400.0	120.0	149.66	1.19	0.05
29	19	20	300.0	400.0	120.0	8.55	0.07	1.23
30	17	20	200.0	400.0	120.0	150.34	1.20	0.05
31	19	21	300.0	400.0	120.0	41.11	0.33	0.25
32	21	22	300.0	400.0	120.0	41.11	0.33	0.25
33	20	22	300.0	400.0	120.0	58.89	0.47	0.18

system. It comprises 33 edges, 22 junction nodes, 10 fundamental circuits, and two supply sources, i.e., one pseudo-circuit. SI units and the Hazen-Williams head loss expression are utilized for this example. Minor loss coefficients are neglected, and the edge resistance is given by

$$\xi = \frac{10.69L}{R^{\sigma}D^{4.87}} \tag{14}$$

in which L is the edge length in meters, R is the Hazen-Williams roughness coefficient, D is the pipe diameter in meters, and the exponent σ is set to 1.852. *Tables*

Table 2. Junction node characteristics

Junction number	Loads (l/sec)	Elevation (m)	Pressure (kPa)
1	−200.0	180.0	455.07
2	−100.0	180.0	447.53
3	−200.0	180.0	448.83
4	−200.0	180.0	456.01
5	−100.0	180.0	438.91
6	−100.0	180.0	436.55
7	−100.0	180.0	443.02
8	−200.0	180.0	447.22
9	−200.0	180.0	449.48
10	−200.0	180.0	432.97
11	−100.0	180.0	429.72
12	−100.0	180.0	427.35
13	−200.0	170.0	505.46
14	−200.0	170.0	503.15
15	−200.0	170.0	506.23
16	−100.0	170.0	501.63
17	−100.0	170.0	501.64
18	−100.0	170.0	503.20
19	−100.0	180.0	395.95
20	−100.0	180.0	395.90
21	0.0	180.0	394.91
22	−100.0	180.0	393.87

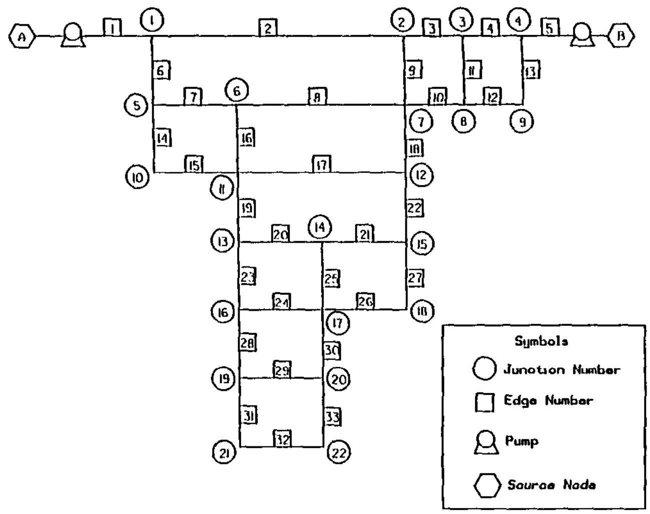

Figure 2. Sample water-conveying network

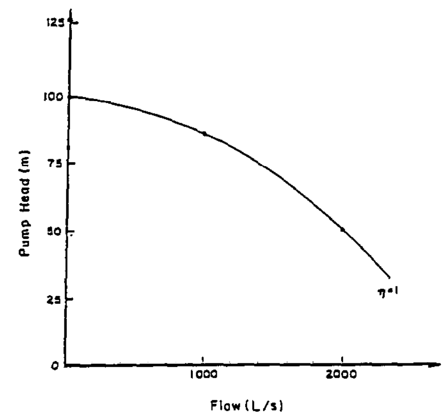

Figure 3. Pump characteristic curve

Table 3. Computational results for supply sources *A* and *B*

Junction number	Source *A* Influence (%)	Source *A* Average age (hr)	Source *A* Maximum age (hr)	Source *A* Minimum age (hr)	Source *B* Influence (%)	Source *B* Average age (hr)	Source *B* Maximum age (hr)	Source *B* Minimum age (hr)
1	100.00	0.09	0.09	0.09	0.00	0.00	0.00	0.00
2	57.75	0.63	0.63	0.63	42.25	0.34	0.34	0.34
3	0.00	0.00	0.00	0.00	100.00	0.12	0.12	0.12
4	0.00	0.00	0.00	0.00	100.00	0.06	0.06	0.06
5	100.00	0.15	0.15	0.15	0.00	0.00	0.00	0.00
6	59.63	0.56	1.02	0.38	40.37	0.70	0.74	0.66
7	29.43	0.75	0.75	0.75	70.57	0.42	0.46	0.38
8	0.00	0.00	0.00	0.00	100.00	0.29	0.32	0.26
9	0.00	0.00	0.00	0.00	100.00	0.15	0.15	0.15
10	100.00	0.25	0.25	0.25	0.00	0.00	0.00	0.00
11	74.67	0.55	1.12	0.44	25.33	0.79	0.83	0.75
12	45.09	0.94	1.59	0.82	54.91	0.62	1.31	0.45
13	74.67	0.59	1.15	0.48	25.33	0.83	0.87	0.79
14	60.77	0.89	1.88	0.61	39.23	0.92	1.59	0.74
15	45.09	0.98	1.64	0.86	54.91	0.66	1.35	0.49
16	73.96	0.76	4.87	0.58	26.40	1.11	4.59	0.89
17	53.94	1.19	2.09	0.77	46.06	1.10	1.81	0.90
18	45.09	1.09	1.75	0.97	54.91	0.77	1.46	0.61
19	73.96	0.81	4.92	0.62	26.04	1.15	4.63	0.94
20	55.02	1.30	6.14	0.82	44.98	1.19	5.86	0.94
21	73.96	1.07	5.17	0.88	26.04	1.41	4.89	1.19
22	62.80	1.40	6.32	0.99	37.20	1.45	6.04	1.12

1 and *2* summarize the pertinent edge and junction node characteristics, respectively. Supply sources *A* and *B* represent wells with energy grades of 160 m. The two well pumps are identical, and the head flow curve for the pumps at full speed ($\eta = 1.0$) is depicted in *Figure 3*. The steady-state flow distribution was obtained by using KYPIPED, a hydraulic network design model developed by Boulos and Wood.[15–17] Their model is based on the network flow solution algorithm described in this paper. The flow algorithm converged in three Newton iterations based on an initial unconstrained velocity distribution of 1 m/sec in all edges. The edge volumetric flow rates and junction node pressures are displayed in *Tables 1* and *2*, respectively.

On the basis of the steady-state flow distribution the proposed algorithms were used to directly determine the source influence characteristics and water age parameters for each junction node. *Table 3* presents the results associated with supply sources *A* and *B*. The calculated results were compared with the results obtained by analyzing the distribution system, employing KYFSI3, a steady-state water quality computer model developed by Wood and Ormsbee.[18] That model is based on a node-ordering methodology combined with an iterative cyclic solution procedure refined with an underrelaxation factor. The solutions obtained by the two methods are numerically identical.

Conclusions

A computer-oriented methodology for carrying out stationary water quality calculations in distribution networks has been presented. The method, which can be applied to circuited, branched, or combination networks of any size, involves a direct solution of successive contingent boundary value problems analytically formulated from mass balance relationships for a variety of water quality parameters. Such capabilities will greatly enhance the ability of engineers to conceive and evaluate efficient and reliable recommendations for monitoring and modeling water quality patterns in multiple-source networks.

Acknowledgments

The authors are grateful to Dr. John J. Vasconcelos of the Department of Computer Aided Engineering at JMM for his many stimulating discussions and suggestions. The authors are also grateful to the anonymous reviewers for their useful comments.

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