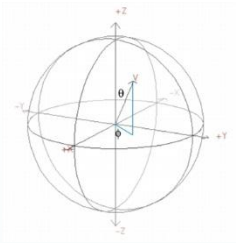

Quantum Computing

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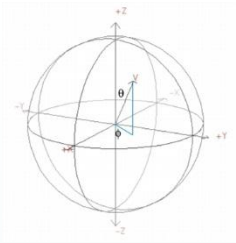
CSC 5446 Presentation

Spring 2015

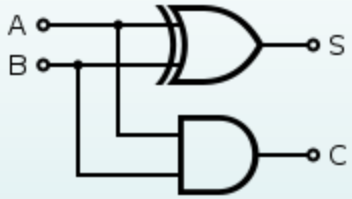
Version 2014.1

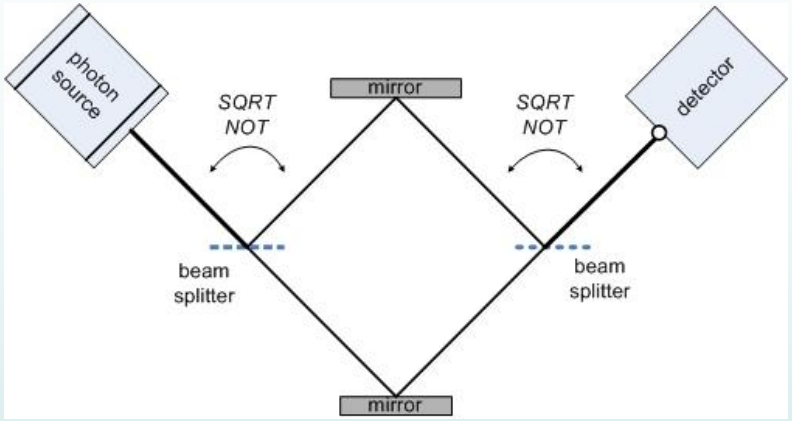


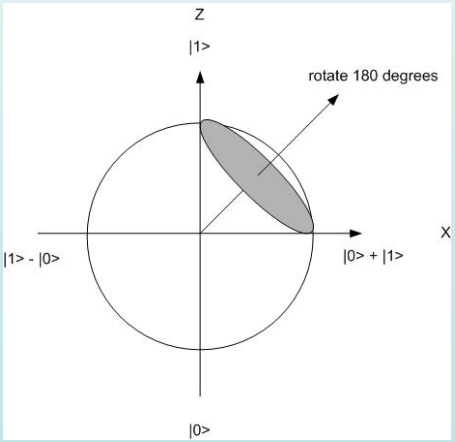
Quantum Computing

Overview of Presentation Topics

Terms:
- Measurement
- Qubit
- Superposition
- etc.



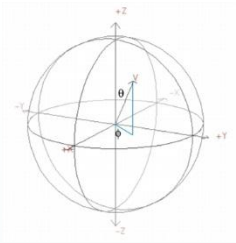




$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$

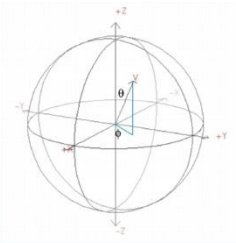
```
Init_state(2)
H(1)
m=[1 0; 0 -1]
U(m,1)
cnot(1,2)
……..
```

Look at quantum computing from C.S. theory point of view:
- Change halting problem?
- Answer to “P versus NP?”
- Impact NP-Complete?



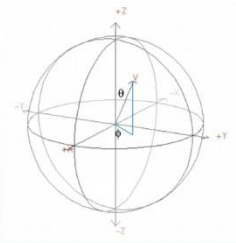
Quantum Computing

- Presentation topics:
  - Terminology of Quantum Computing (“QC”)
  - Quick comparison of the digital, analog and QC technologies
  - List of QC algorithms, and a look at one algorithm
  - QC “reality check”
  - QC impact on Computer Science Theory
  - Wrap up and questions
- Presentation omissions:
  - Quantum encryption/security, teleportation
  - Universal quantum logic gates
  - Higher-level mathematics of quantum physics
  - Quantum error detection/correction
  - Quantum annealing



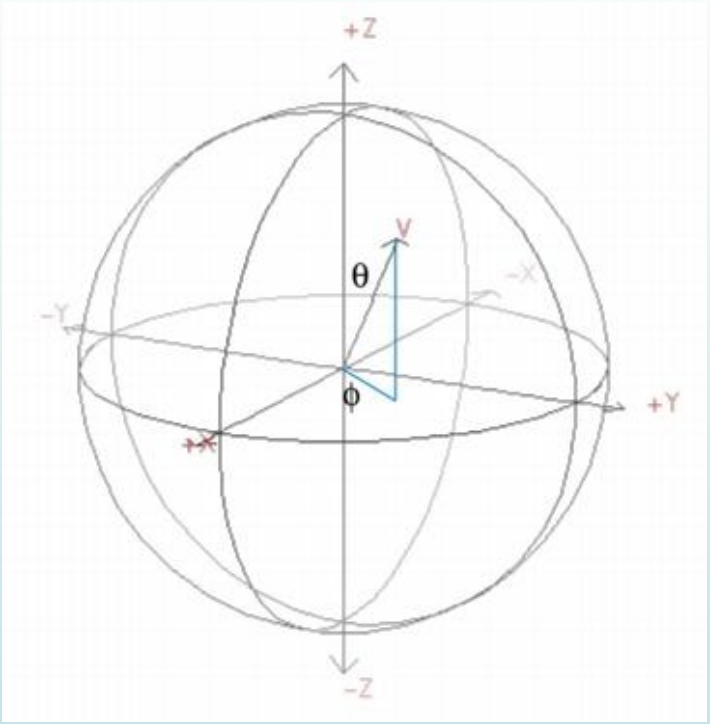
Quantum Computing, Some Basic Questions

- In the abstract, can Quantum Computing (“QC”) perform processing/computations?
- Is QC viable? Similar questions: Is QC useful? Does the use of QC ‘make sense?’
- Are there quantum computers available today?
- Does QC have a significant impact on C.S. with the “P versus NP” problem or with the solution of “NP-Complete” problems?

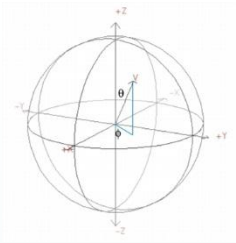


← This is a “Bloch Sphere.”

- The Bloch Sphere is a QC presentation showing the “the pure state space of a 1 qubit quantum register.”
- A qubit is a quantum bit.
- The vector to V is length 1 and its X, Y, Z coordinates provide the ‘probability amplitudes’ for the qubit.
- Qubit also shown in $[0\ 1]^T$ or |1> format.

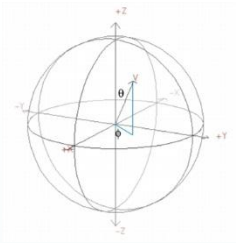


Note: We will cover QC terms, like “qubit,” in just a few more slides.



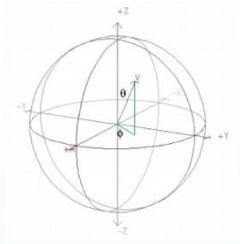
Quantum Computing

- What is Quantum Computing?
  - It is computation done using quantum hardware components.
  - The Heisenberg Uncertainty Principle applies:
    - Certain pairs of physical properties, like position and momentum, cannot both be known to arbitrary precision.
    - Measurement can cause state changes, for example the collapse of a wave function.
    - Before measurement we speak of probabilities, after measurement we speak of binary values: |0> and |1>. The QC behavior is “irreducibly random.”



Some Key People in QC

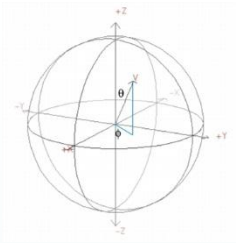
- Richard Feynman - Physicist
  - Presented a solid justification for QC in his 1981 paper.
  - Saw that the simulation of quantum mechanics could be a high-value use of QC.
- David Deutsch - Physicist
  - Championed the *multiverse* (multiple universe, many-worlds) view of *superposition*.
  - Produced some of the first significant QC algorithms. We will briefly look at his first algorithm.



Terminology of QC

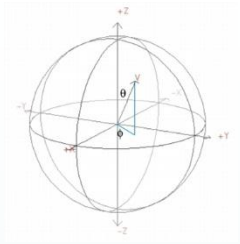
(and some basic quantum mechanics)

- Qubit
  - A quantum “bit” of information.
  - Often based on particle’s polarization or spin.
  - When measured, the qubit will have one of two values.
  - Before measurement, qubit state is viewed as a complex number, a “probability amplitude.”
  - Common ways to express or view a qubit’s state:
    - Bloch sphere
    - 2 x 1 matrix view; examples: $[0 , 1]^T$, $[1, 0]^T$, $[SQRT(2)/2, SQRT(2)/2]^T$
    - Dirac “bra-ket” notion; “ket” examples: $|1>$, $|0>$, $\alpha|0> + \beta|1>$
  - Note: one particle can represent two qubits by utilizing both polarization and spin.



Describing Qubit Values

- With Bloch Sphere
  - Value |1> is triplet: X: 0, Y: 0, Z: -1
  - Value |0> is triplet: X: 0, Y: 0, Z: 1
  - Note: These X, Y, Z values are *probability amplitude* components.
- With Matrices
  - |0> is $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
  - |1> is $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
  - H(|1>) is $\begin{pmatrix} .5 & -.5 \\ -.5 & .5 \end{pmatrix}$ Note: H() is a Hadamard gate and this value is in “superposition.”



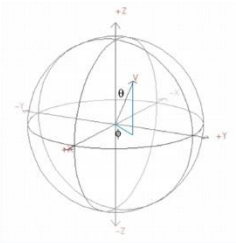
Terminology of QC

(and some basic quantum mechanics)

Needed: *"A willing suspension of disbelief."*

This is especially true for *Superposition* and *Entanglement*.

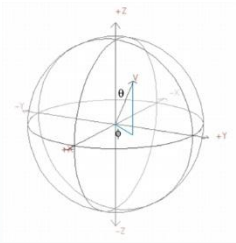
- Quantum Behaviors
  - No-Cloning Theorem
  - Probability Amplitudes
  - Reversibility
- Qubit States
  - Measurement
  - Transformations
  - *Superposition*
  - *Entanglement*
  - Decoherence



Terminology of QC

(and some basic quantum mechanics)

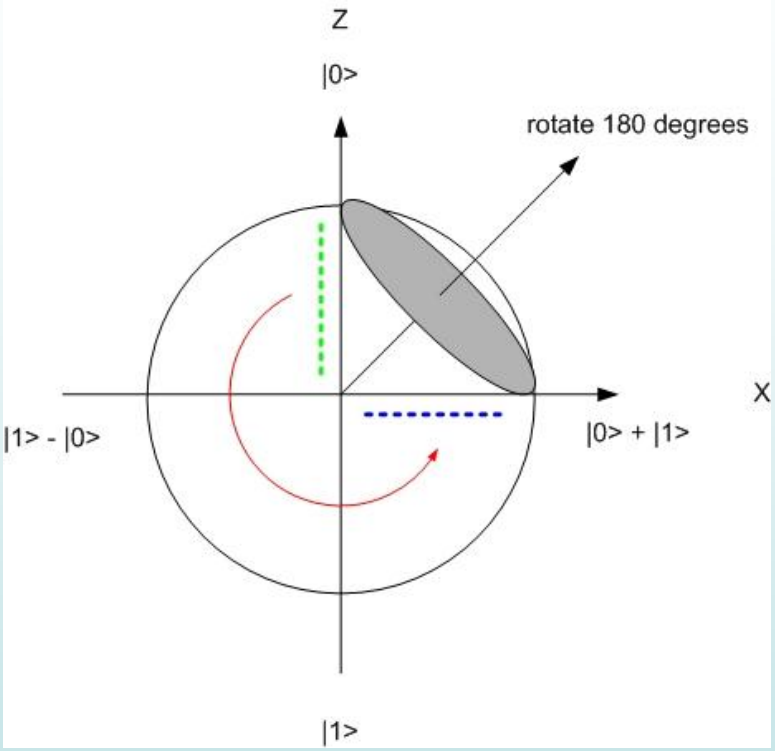
- Measurement
  - Observing the state of the qubit.
  - Measurement always results in one of two results: |0> or |1>.
  - Result depends on the probability amplitudes.
  - Measurement might change the state of the qubit.
- Probability Amplitudes
  - Complex number associated with a qubit.
  - These amplitudes can be negative!
  - Summing the squares of the absolute values = 1.
  - In simple terms: Probability amplitudes determine the behavior/value of the qubit.



Terminology of QC

(and some basic quantum mechanics)

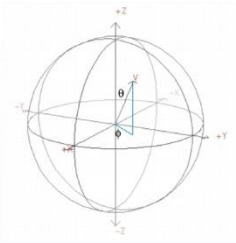
- Transformations/Evolutions
  - Application of a “rotation” or phase shift to a qubit.
  - This is not measurement!
  - Can be viewed as matrix multiplication.
- Quantum Programming is done with these transformations!



“Hadamard” Transformation.

Const. Value | Trans Matrix | Qubit Value

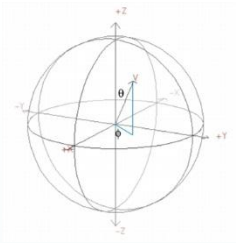
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$



Terminology of QC

(and some basic quantum mechanics)

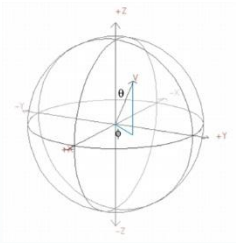
- Superposition (“multiverse”)
  - A phenomenon where an object exists in more than one state simultaneously.
  - Enables qubits to have simultaneous values.
  - In simple terms: In this state, a qubit can be |0> and |1> at the same time (but we cannot view this).
- Entanglement
  - The quantum states of the involved objects are linked together so that one object can no longer be adequately described without full mention of its counterpart.
  - In simple terms: Changing one object changes the other; they are “linked” even if physically separated.



Terminology of QC

(and some basic quantum mechanics)

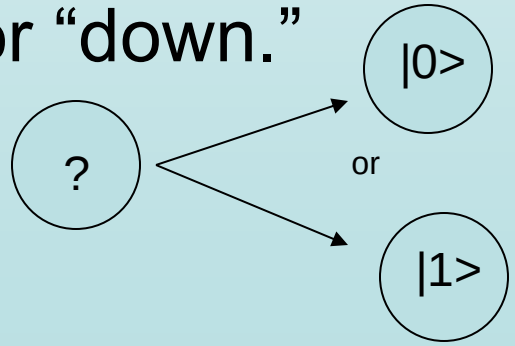
- Decoherence
  - Untangling of quantum states to produce a single reality.
  - This “untangling” or collapse may occur prematurely.
  - In QC, decoherence may require error correction.
  - In simple terms: Quantum state collapses to one value. Can be viewed as premature measurement.

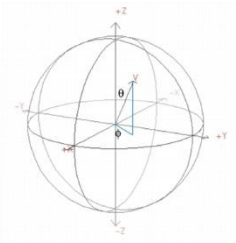


Terminology of QC

(and some basic quantum mechanics)

- Polarization and Spin
  - These are two, of many, quantum characteristics that can be used to encode information.
  - Polarization refers to the polarity of light.
  - Spin refers to a measurable characteristic of quantum particles/waves, usually expressed, at measurement, as “up” or “down.”

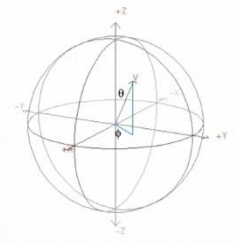




Terminology of QC

(and some basic quantum mechanics)

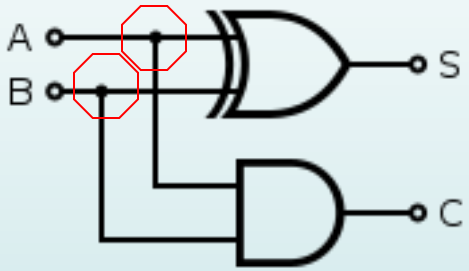
- No-Cloning Theorem
  - Qubits cannot be directly copied.
  - In simple terms: cannot code “qubit1 = qubit2;” in a QC environment.
  - How to debug errors, if can’t copy or measure intermediate results??
- Reversibility (Landauer’s principle)
  - Quantum actions are reversible.
  - In simple terms: Quantum logic gates have an equal number of inputs and outputs.
  - Note: Measurement is not reversible.



Computer Hardware/Circuits

(Digital and Analog Examples)

Digital Computer Circuit

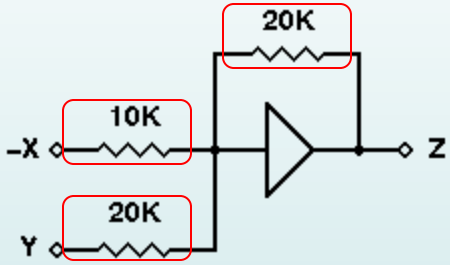


Half Adder:

S = A xor B

C = A and B

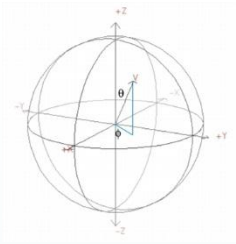
Analog Computer Circuit



Voltage: Z = 2X-Y

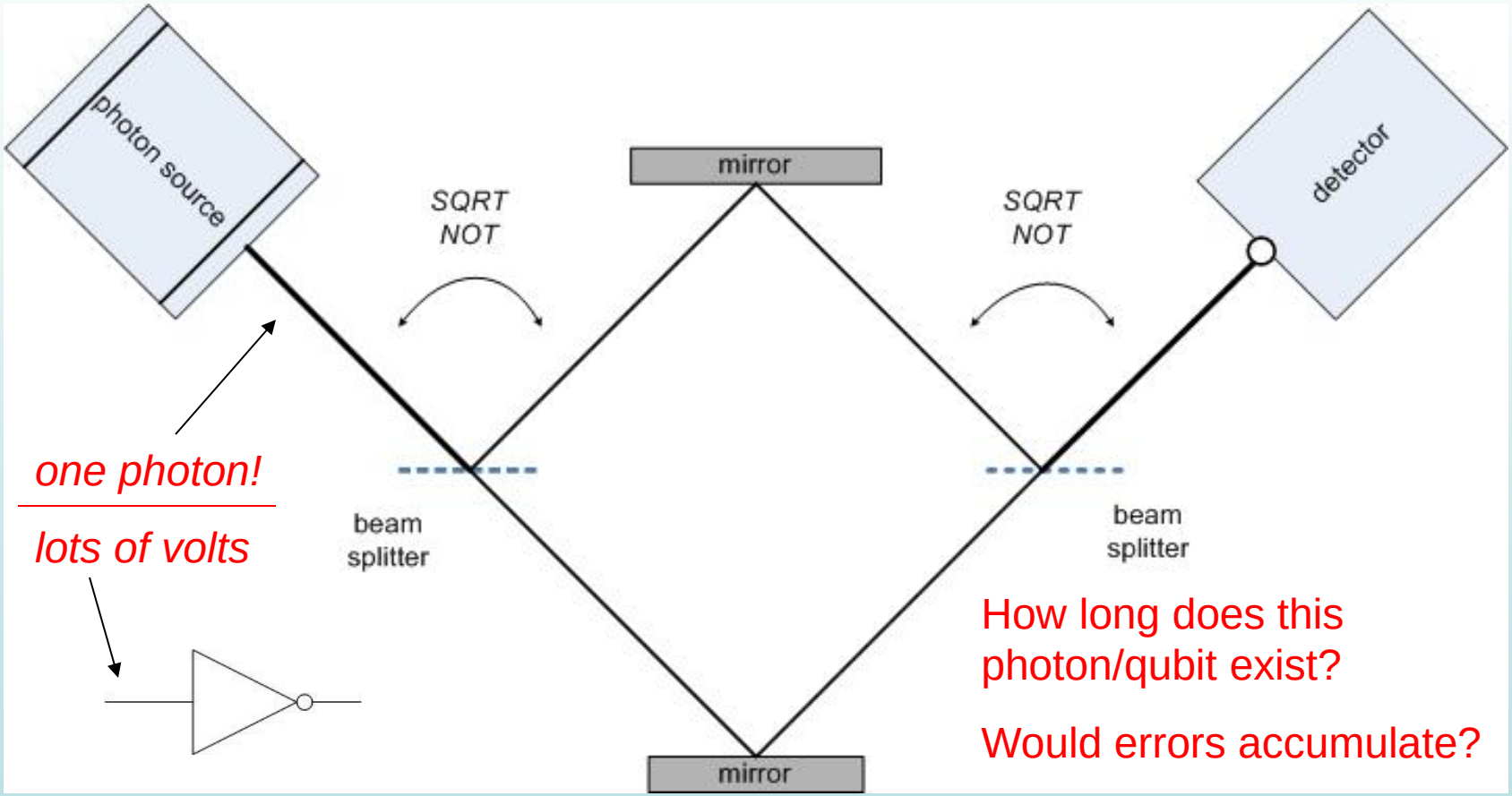
Comments:

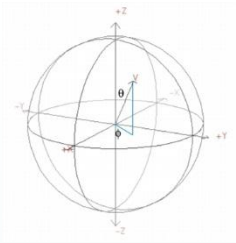
- Half Adder clones the values of pin A and pin B.
- Analog circuits tend to accumulate errors.



Computer Hardware/Circuits

QC Hardware: NOT “Gate” (Pauli-X)





Values in Bits and Qubits

3-Bit Register	
value	likelihood
000	0
001	0
010	1
011	0
100	0
101	0
110	0
111	0

3-Qubit Register	
value	likelihood
000	C_0
001	C_1
010	C_2
011	C_3
100	C_4
101	C_5
110	C_6
111	C_7

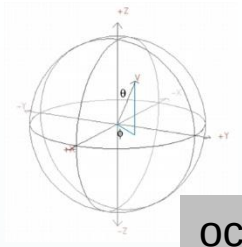
Feynman's Point: Quantum values use huge amounts of "digital" resources:

- 64-bits: ~ 8 bytes
- 64-qubits: ~2^{64} complex numbers.
- A digital calculation on two 64-qubit values in superposition is 2^{128} calculations!

value is 010;
bit fits in < <u>one byte</u>

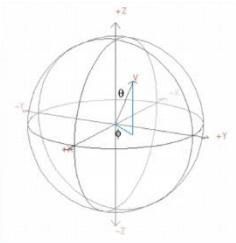
probability of any pattern is C_P;
qubit fits in 8 complex numbers
(~64 bytes)

(18,446,744,073,709,551,616 complex numbers = one 64-qubit register!)



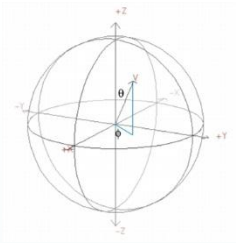
Describing Qubit Values

```
octave-3.2.3.exe:3:C:\Octave\3.2.3_gcc-4.4.0\bin\Quack!
> init_state(1)
octave-3.2.3.exe:4:C:\Octave\3.2.3_gcc-4.4.0\bin\Quack!
> print_dm(1)
   1   0
   0   0
octave-3.2.3.exe:5:C:\Octave\3.2.3_gcc-4.4.0\bin\Quack!
> init_state(3)
octave-3.2.3.exe:6:C:\Octave\3.2.3_gcc-4.4.0\bin\Quack!
> print_dm(3)
   1   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0
octave-3.2.3.exe:7:C:\Octave\3.2.3_gcc-4.4.0\bin\Quack!
>
```



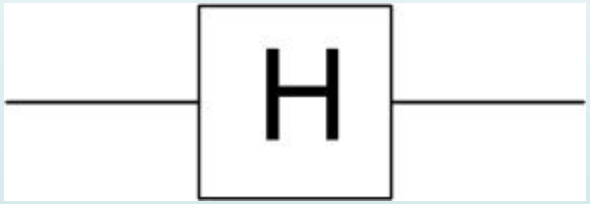
Quantum Hardware

- The quantum hardware qubits are maintained in different ways, here are two:
  - Particle stream
    - Particle moves through quantum gates between a source and detector.
    - Qubit is a characteristic of the particle: polarization or spin.
  - Quantum dot
    - Particle is stationary.
    - Qubit is represented by a characteristic of the particle: polarization, electron orbit or spin.

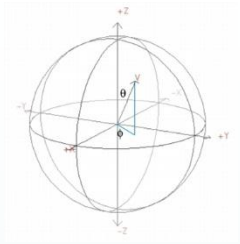


Quantum Logic Gates

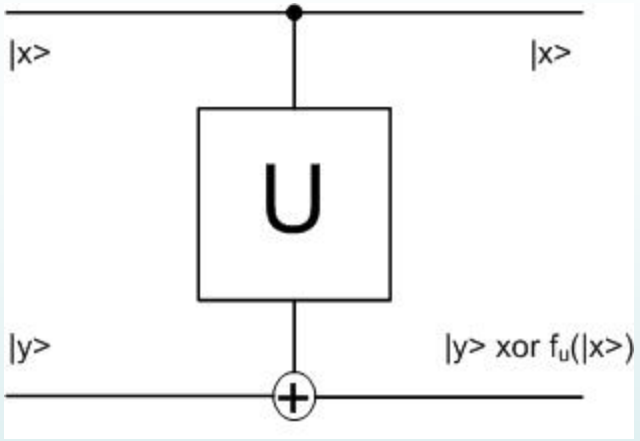
- SQUARE-ROOT-OF-NOT
  - We saw this earlier: QC Hardware: NOT “Gate.”
  - Evolution: $1/(2^{1/2})\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$
  - Done twice is a NOT operation.
- Hadamard (“H gate”), symbol:



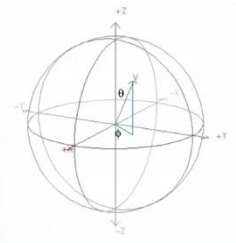
  - We will see this used.
  - Puts a qubit into a state of superposition.
  - Evolution: $1/(2^{1/2})\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$
  - Done twice returns original state.



Quantum Logic Gates

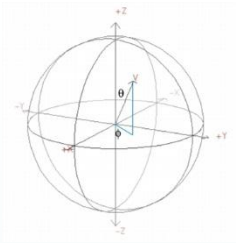


- C-NOT, symbol:
  - We will see this used.
  - |x> controls behavior of the output.
  - Observation: If |y> is zero then output is f_u(|x>); ‘xor’ operation acts like a copy.
  - But wait, what about the *No-Cloning Theorem*?
    - This is not actually a clone or copy.
    - |x> output is now entangled with |y> !!



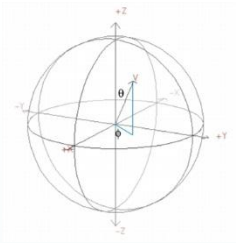
Quantum Logic Gates

- Some other common quantum gates:
  - Toffoli:
    - Similar to C-NOT, but with two control inputs
    - 3 input and 3 outputs
    - outputs: |x>, |y>, and (|(z xor (x and y))>
  - Pauli-X, Pauli-Y, Pauli-Z:
    - Rotations about X, Y, or Z axis.
    - Pauli-X is a “NOT” operation.



Famous QC Algorithms

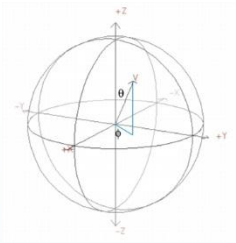
- Deutsch's Algorithm (1985/1992)
  - First "useful" QC algorithm.
  - Very contrived financial problem.
  - We will see this algorithm digitally simulated.
- Grover's Search Algorithm (1997)
  - Searches an unordered list in $O(N^{1/2})$ time.
  - Non-quantum algorithm takes O(N/2) time.
- Shor's Factoring Algorithm (1994)
  - Integer factoring of numbers.
  - Uses periodicity ("mod") and Fourier Transform.
  - In 2001, demonstrated by IBM researchers using NMR-based hardware with 7 qubits.



Deutsch's Algorithm

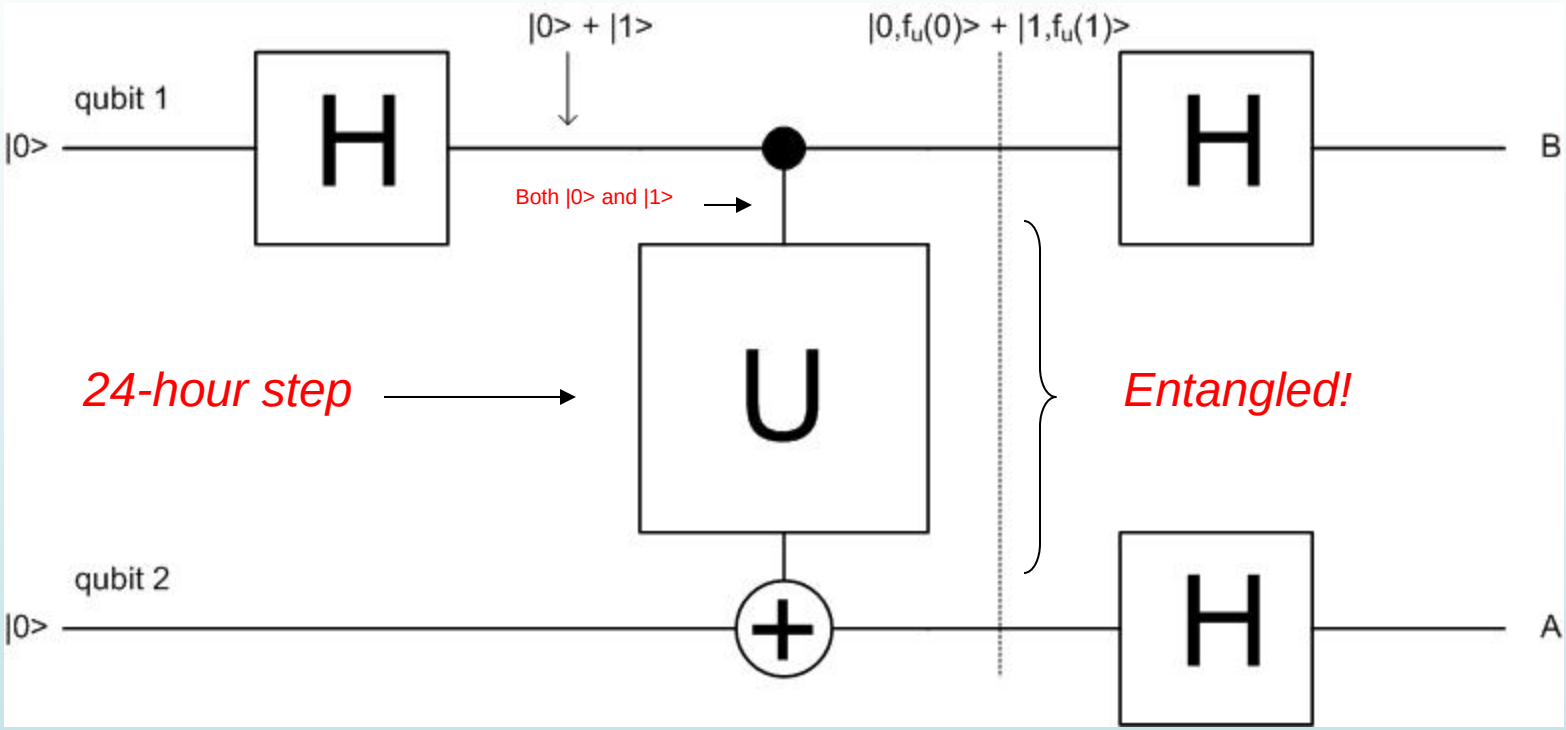
(Briefly look at and simulate Deutsch's Algorithm)

- The contrived problem:
  - Have a financial "algorithm" that runs for 24 hours; run it twice to get a recommendation.
  - The final result is Exclusive-OR (XOR) of the two runs, each with a different input.
  - A digital computer needs 48 (2 * 24) hours.
  - Results are needed in 24 hours.
  - If the binary results of each run match, we act.
  - "Act" could be buy stock, sell stock, etc.



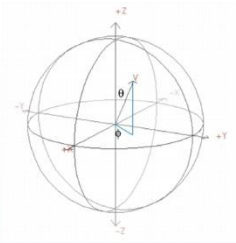
Deutsch's Algorithm

(Perhaps better titled: Deutsch's Circuit)



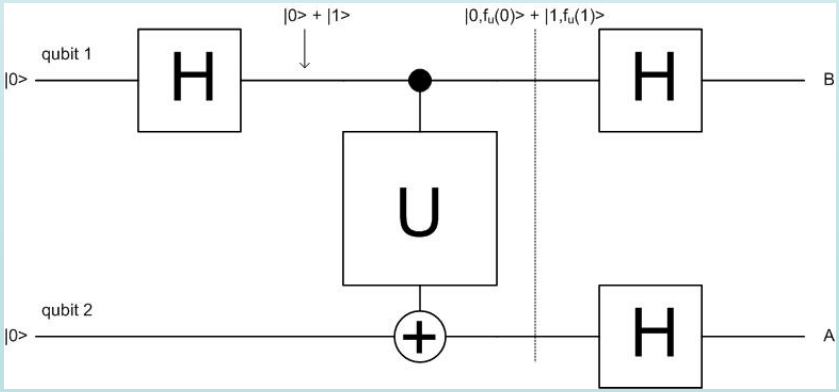
A: Measure first; if zero then inconclusive else measure B.

B: Measure second; if 0 then f is same; if 1 then f is different.

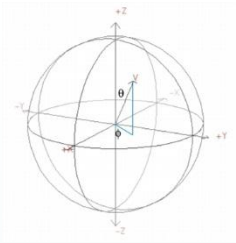


Deutsch's Algorithm

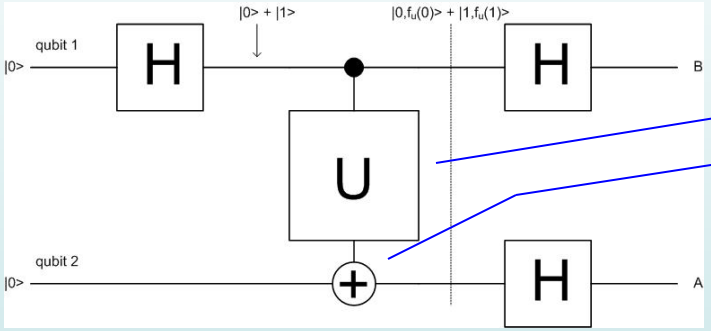
- My goal: To simulate the execution of Deutsch's algorithm. /* my quantum "Hello world" program… */
- Done using MatLab (Octave) and "Quack!"
- Quack! is a quantum computer simulator for MATLAB, by Peter P. Rohde, University of Queensland, Brisbane, Australia.
- This simulator made it possible to get some "hands on" experience with quantum computer programming.




```
Init_state(2)
H(1)
m=[1 0; 0 -1]
U(m,1)
cnot(1,2)
……..
```

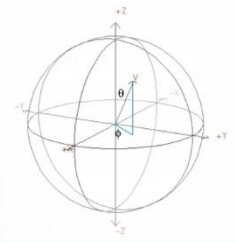


Quack! & Deutsch's Algorithm



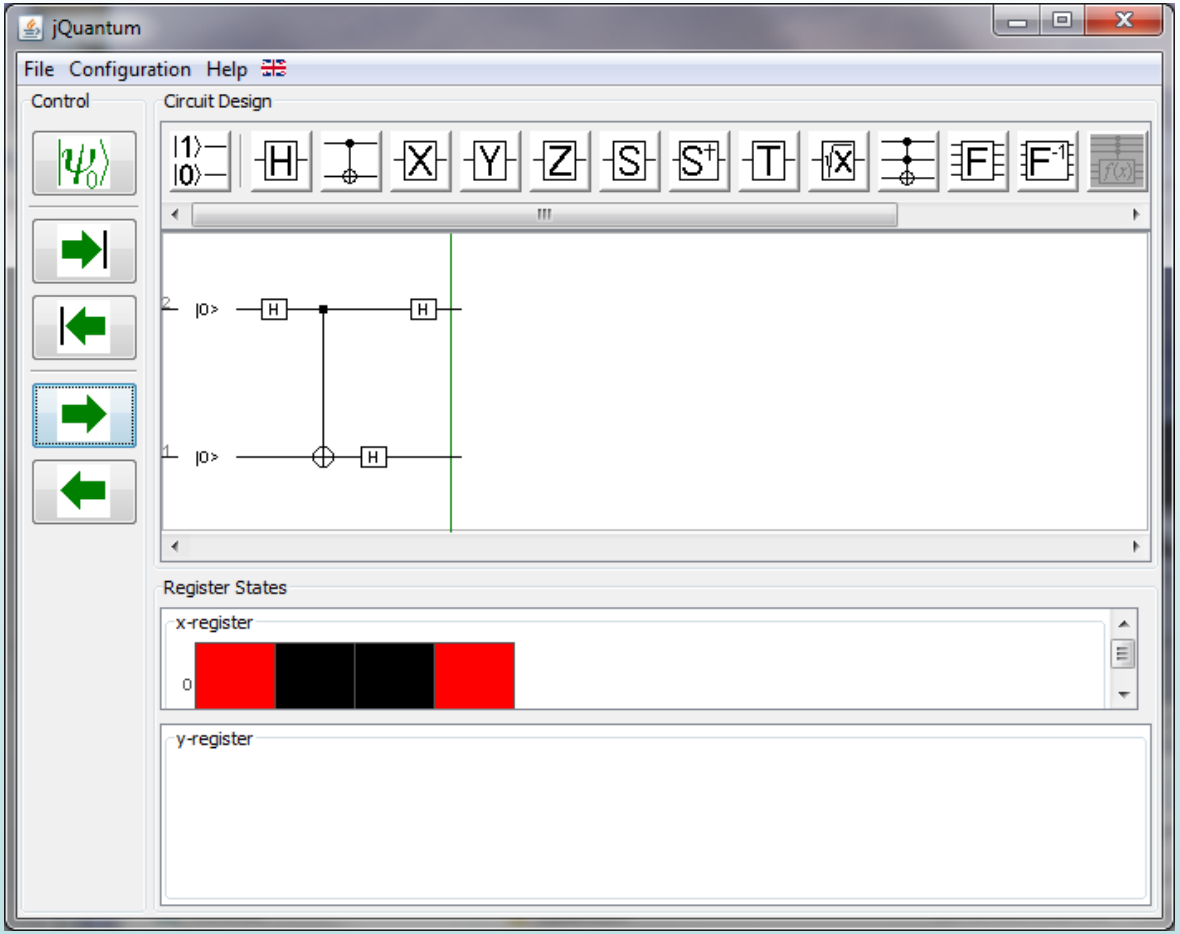

```
# Dcase3.m:
init_state(2)
H(1)
# f(i) = 0
m=[1 0 ; 0 -1]
U(m,1)
cnot(1,2)
H(1)
H(2)
disp('bit2:')
Z_measure(2)
disp('bit1:')
Z_measure(1)
```

```
> source Dcase3.m
m =
   1   0
   0  -1
bit2:
ans =  1    is: |0>
bit1:
ans = -1    is: |1>
```

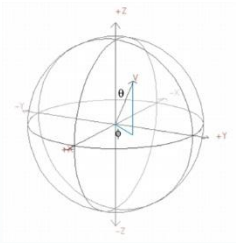


jQuantum

http://jquantum.sourceforge.net/



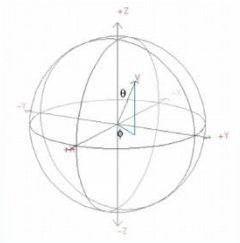
As of 2014, "Quack!" is no longer available, here is a look at: "jQuantum"



Feynman's QC Vision

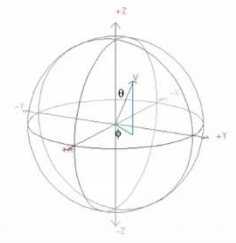
- Solving Quantum Mechanics problems with Quantum Computing.
- What's the problem? The "tensor product;" the explosion of terms.
- In Octave (like MATLAB), using "Quack!"
  - init_state(12) creates 12 qubits.
  - init_state(16) exhausts memory!
- But how much measurable "information" is there in N qubits? Exactly N bits!

(18,446,744,073,709,551,616 complex numbers = one 64-qubit register!)



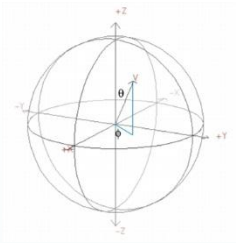
QC Personal Observations

- Before we talk about QC and Computer Science, here are my personal observations:
  - QC algorithms are not easy to understand and produce.
  - QC does not appear to scale well.
    - Scaling from 4 to 32 qubits is not easily done.
    - Quantum gates in serial might accumulate errors.
  - QC is probability-based, so it may not be sufficiently precise.
  - QC appears useful mainly in very specialized cases.
    - Creation of “Oracles.”
    - Simulation of quantum systems.
  - QC is likely to become real – but not anytime soon.
    - ENIAC, early digital computer (1946), vacuum tube-based, estimated MTF: 10 minutes; reality: nonfunctional about half the time. The invention of the transistor enabled the practical digital computer.
    - Observation: NSA, RSA, etc. do not appear concerned about QC breaking current security schemes.



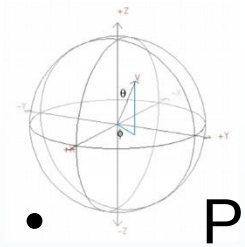
“Hype” associated with QC

- There is ample “hype” associated with QC.
  - The notion of qubit registers holding “infinite information” is expressed, common in the popular press. This notion is wrong! N qubits hold N bits.
  - The multiverse concept appears in movies and TV, for example as parallel universes in *Star Trek, etc*.
    - Superposition is on a very small scale.
    - Decoherence tends to occur with any environmental interactions.
  - Some people suggest that QC will show P=NP or solve NP-Complete problems. This is our next topic.



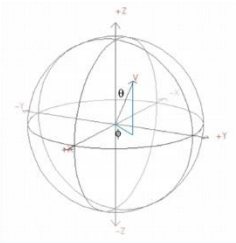
QC Impact on CS Theory

- Does quantum computing impact Computer Science theory?
  - Change the “halting problem?”
  - Have impact on the “P versus NP” problem?
  - Make NP-Complete problems “easy” to solve?
- Before looking into these questions, a review of some basic CS theory.



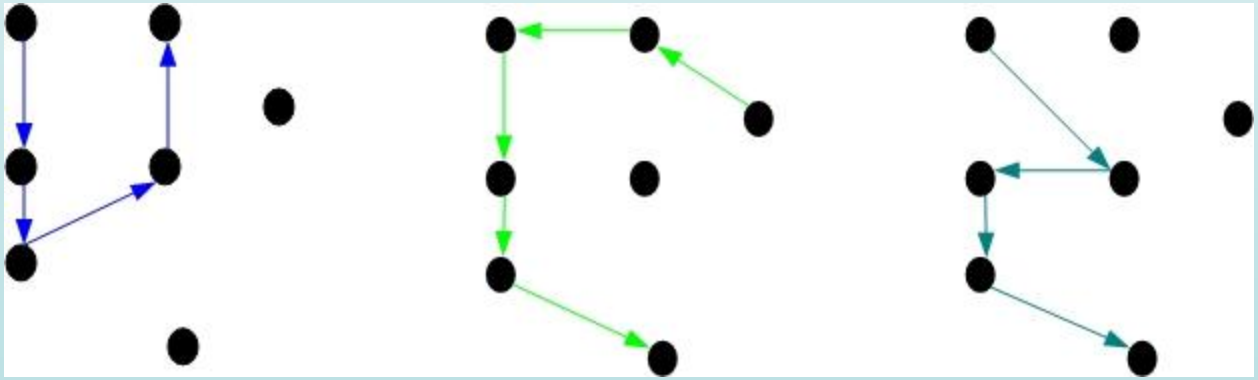
P and NP Problems

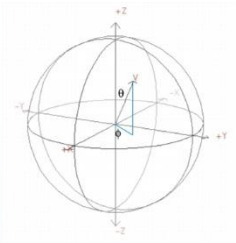
- Problems in class P (Polynomial time)
  - The class of problems with efficient solutions.
  - Solvable in polynomial time, (run time "N^2" -- not "2^N").
  - Example: Find a integer in an un-ordered list of integers.
- Problems in class NP (Nondeterministic Polynomial time)
  - The class of problems that have efficiently verifiable solutions.
  - Finding solutions is not polynomial, verifying the solution is.
  - Example: "subset-sum" problem.
    - Given a set of integers, does some subset add up to zero?
    - Example data: (-2, -3, 15, 14, 7, -10)
- "P=NP" means: For problems with efficiently verifiable solutions we can also find efficient solutions. Restated: every problem whose solution is rapidly verified by a computer is also rapidly solved by a computer. No one has yet proven this notion, "P=NP", to be true or false.



NP-Complete Problems

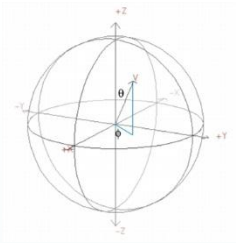
- NP-Complete problems
  - NP-Complete problems are NP problems (easy to verify, hard to solve) and are transformable into each other.
  - The satisfiability (SAT) logic problem can be transformed into any NP-Complete problem. The satisfiability problem is at least as hard as any other problem in NP.
  - If a NP-Complete problem can be solved quickly then so can every problem in class NP!
- Example: Traveling salesperson
  - Asks for the shortest distance through a set of cities.
  - Here are 3 possible solutions, each with 7 cities. Imagine 10,000 cities!





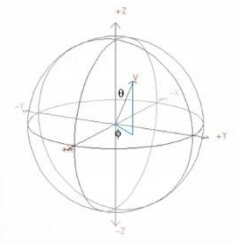
QC & P/NP/NP-Complete

- "Halting Problem" and QC
  - HP is not a constraint because of limited computational speed; it is due to a basic limit in the nature of computation itself.
  - My view: No impact.
- QC impact on "P = NP?"
  - Shor's Factoring Algorithm does appear to break into the class of NP problems.
  - For some classes of problems, like Feynman's quantum mechanics issues, it appears that QC can drastically increase computational speed.



QC & P/NP/NP-Complete

- Impact on solving NP-Complete problems.
  - Many NP-Complete problems are non-optimally solved today using heuristics & hybrid solutions based on statistics, trial and error, and brute force. Today there often exist “good enough,” non-optimal solutions.
  - My view:
    - It is not clear that QC brings forth something new for solving the NP-Complete problems.
    - In addition, solving problems in terms of manipulating probability amplitudes makes for very complex algorithms.
    - Trying to use classic digital gates, circuits, and algorithms causes an “explosion” in the number qubits, due to non-cloning and reversibility, and this makes classic digital processing in QC impractical. Likely that practical quantum computers will be linked with digital computers.



Halting Problem

- Nothing to do with QC: Will these halt?

```
n = 2
for i=1 to infinity;
   for j=1 to infinity;
      for k=1 to infinity;
         if (i^n == j^n + k^n) exit;
```

```
n = 3
for i=1 to infinity;
   for j=1 to infinity;
      for k=1 to infinity;
         if (i^n == j^n + k^n) exit;
```

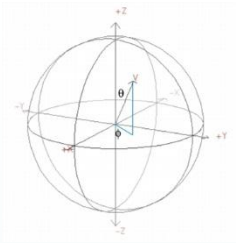
Assume all i, j, k : first 1,000 values are tried, then values < 10^6 are tried, etc. What problem/theorem are these related to? Fermat's Last Theorem!

```
n = 3
for i=1 to 1000;
   for j=1 to 1000;
      for k=1 to 1000;
         if (i^n == j^n + k^n) exit;
```

```
n = 3
for i=1 to 1000000;
   for j=1 to 1000000;
      for k=1 to 1000000;
         if (i^n == j^n + k^n) exit;
```

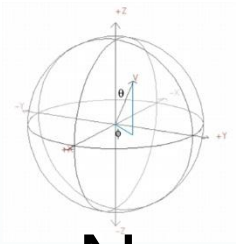
```
n = 3
for i=1 to 10^9;
   for j=1 to 10^9;
      for k=1 to 10^9;
         if (i^n == j^n + k^n) exit;
```

…



QC in 2015

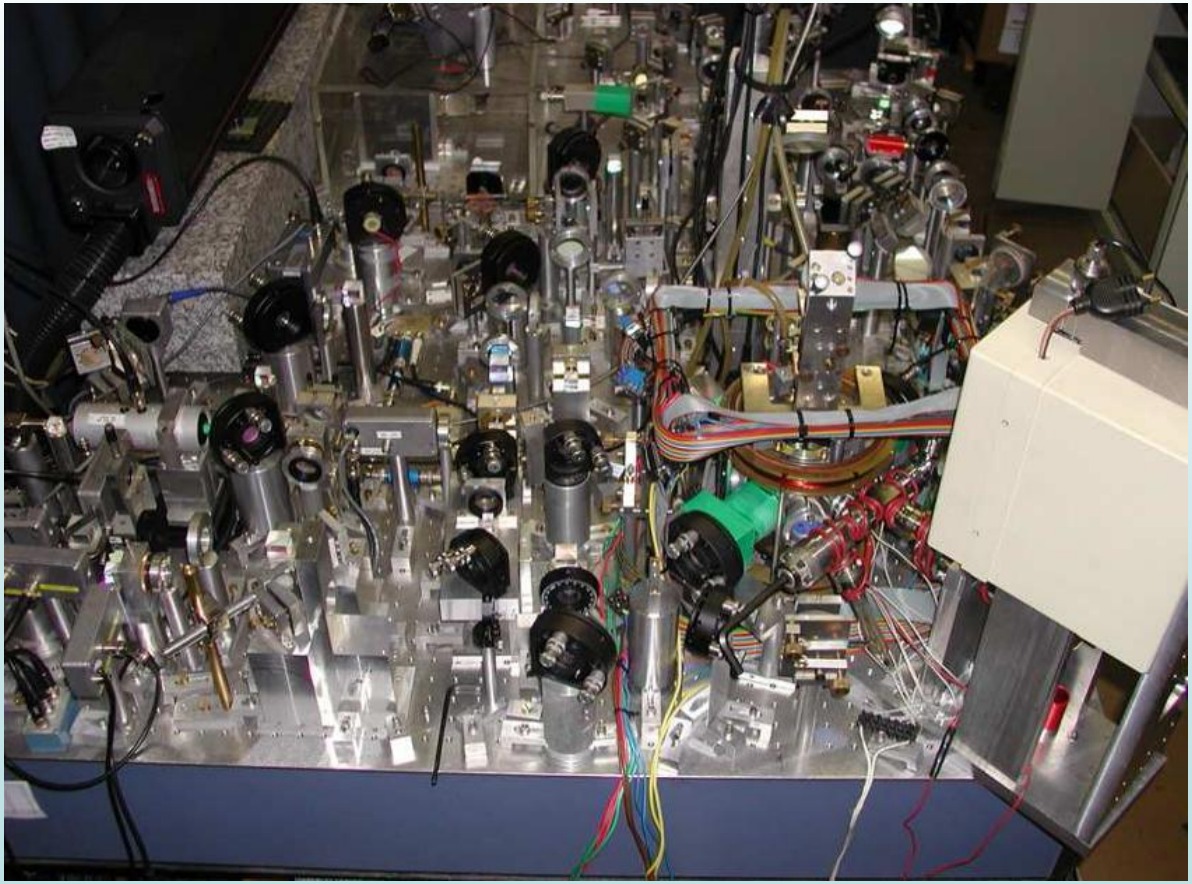
- What has changed recently with QC?
  - When I walk through *Best Buy,* I do not see any quantum computers.
  - To the best of my knowledge, no new major quantum computer companies have formed.
  - But, progress has occurred.
  - Let's review 2010, 2012, and then look at 2015.

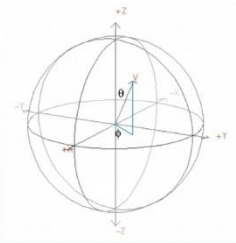


QC 2010 Realities

- No commercial QC computers existed!

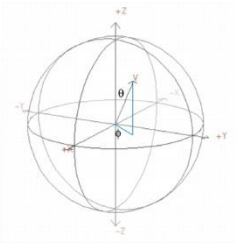
2010 lab picture of a large quantum circuit using laser-beam based gates:



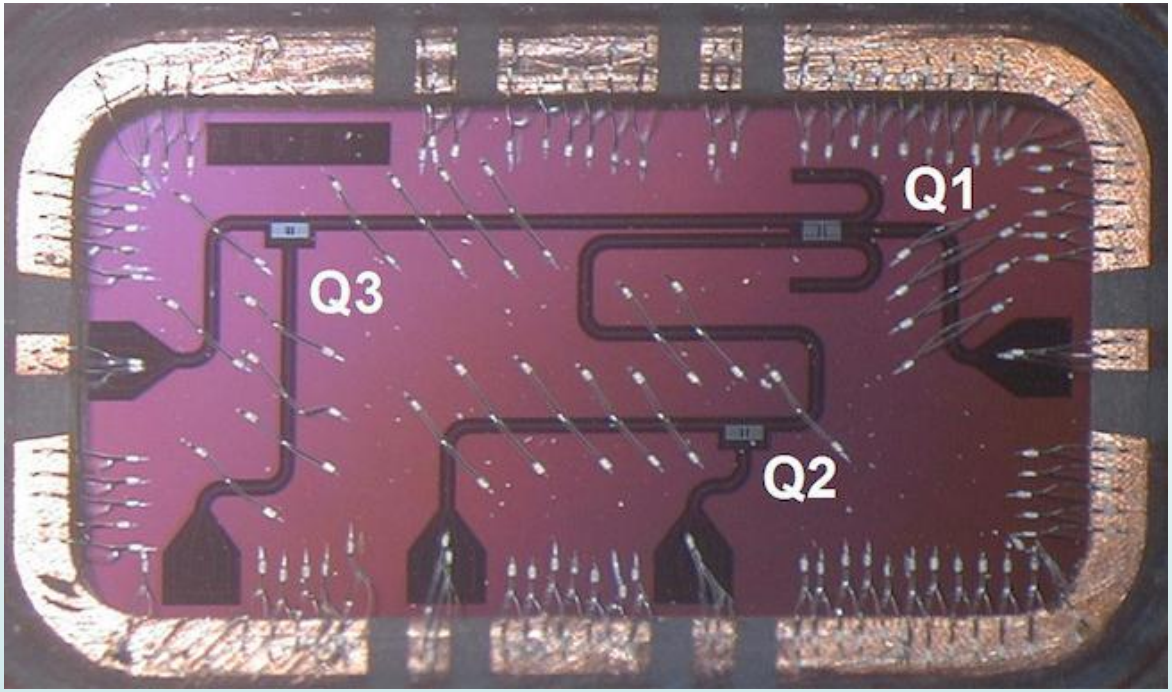


QC Realities

- 2010 Status
  - Commercial QC computer attempt in 2007
    - D-Wave Company, claimed 16 qubits.
  - A register of 8 to 12 qubits is “noteworthy.”
  - Shor actually factored 15 into 3 and 5!
  - Loosing coherence at 1.2 seconds is news.
- 2012 Status
  - Factored 143 (11*13) using quantum computing.
  - D-wave: claims 84 qubits. Sells a 128-qubit for $10M.
  - IBM has produced a three-qubit chip.
  - Single atom transistor, has quantum behaviors.

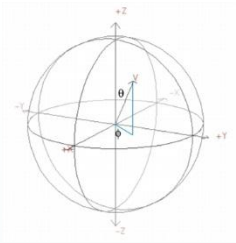


IBM 3-qubit Chip



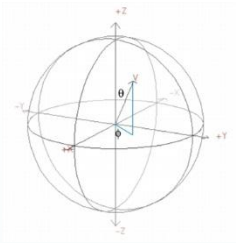
“IBM’s team has also built a “controlled NOT gate” with traditional two-dimensional qubits, meaning they can flip the state of one qubit depending on the state of the other. This too is essential to building a practical quantum computer, and Steffen says his team can successfully flip that state 95 percent of the time — thanks to a decoherence time of about 10 microseconds.”

source: http://www.wired.com/wiredenterprise/2012/02/ibm-quantum-milestone/



QC 2015 Realities

- Commercial QC computer 2014/2015
  - D-Wave Company, claims 128 qubits.
  - Working with Lockheed Martin and more recently NASE and Google.
  - Direct quote: “D-Wave's architecture differs from traditional quantum computers (none of which exist in practice as of today) in that it has noisy, high error-rate qubits. It is unable to simulate a universal quantum computer and, in particular, cannot execute Shor's algorithm.”
- But: uses “quantum annealing”.



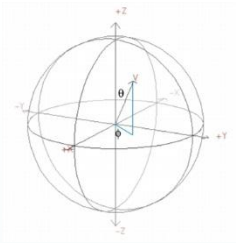
D-Wave Background

- The D-Wave company is controversial.
  - Geordie Rose is CEO.
  - Canadian company; in past, Google invested money.

"Geordie Rose has a Ph.D. in quantum physics, but he's also a world champion in Brazilian jiu-jitsu and a Canadian national champion wrestler. That may seem like an odd combination, but this dual background makes him the perfect fit for his chosen profession."

"But Rose keeps fighting. In May, D-Wave published a paper in the influential journal *Nature* that backed up at least some of its claims. And more importantly, it landed a customer. That same month, mega defense contractor Lockheed Martin bought a D-Wave quantum computer and a support contract for $10 million."

source: http://www.wired.com/wiredenterprise/2012/02/dwave-quantum-cloud/

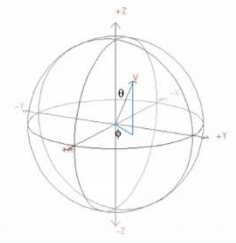


Quantum Annealing

Definition: "**Quantum annealing** (QA) is a metaheuristic for finding the global minimum of a given objective function over a given set of candidate solutions (candidate states), by a process using **quantum** fluctuations." [WIKI]

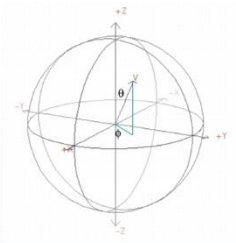
Quantum annealing is NOT the type of Quantum Computing we have just discussed. Quoted from the Wiki article:

D-Wave's architecture differs from traditional quantum computers (none of which exist in practice as of today) in that it has noisy, high error-rate qubits. It is unable to simulate a universal quantum computer and, in particular, cannot execute Shor's algorithm.



QC Realities

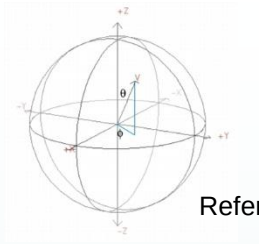
- Again, Shor actually factored 15 into 3 and 5!
- A qubit taking 1.2 seconds to loose coherence makes for a significant journal article.
- Due to decoherence, error correction remains a major issue.



Wrap Up

(Thoughts about Basic Questions)

1. In the abstract, can Quantum Computing ('QC') perform processing/computations?
   Yes
2. Is QC viable? Similar questions: Is QC useful? Does the use of QC 'make sense?'
   2010: Perhaps 2015: Perhaps++ ☺
3. Are there quantum computers available today?
   No (2010); Yes (2012), Bigger (2014), but….
4. Does QC have a significant impact on C.S. such as the "halting" problem?
   What do you think?



QC References

References:

[] Aaronson, S "The Limits of Quantum", *Scientific American*, pp 62-69, March 2008.

[] Bacon, D., Van Dam, W. "Recent Progress in Quantum Algorithms," *Communications of the ACM*, vol 53, no 2, pp. 84-93, February 2010.

[] Bigelow, K. Analog Addition [Internet]. www.play-hookey.com; 3/8/2010. Available from: http://www.play-hookey.com/analog/analog_addition.html.

[] Brown, J. *The Quest for the Quantum Computer*, New York: Touchtone/Simon and Schusterm, 2001.

[] Feynman, R. "Simulating Physics with Computers," *International Journal of Theoretical Physics*, vol 21, no 6/7, 467-488, 1982.

[] Fortnow, L. "The Status of the P versus NP Problem," *Communications of the ACM*, vol 52, number 9, pp. 78-86, September 2009.

[] Morton, J and others "Soid-state quantum memory using the 31P nuclear spin," *Nature*, vol 455, pp 1085-1087, Octobert 2008.

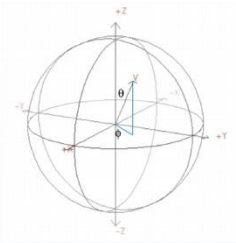
[] Milburn, G. *The Feynman Processor*, New York: Helix Books, 1998.

[] Yanofsky, N., Mannucci, M. *Quantum Computing for Computer Scientists*, Cambridge: Cambridge University Press, 2008.

[] Wikipedia contributors, Adder (electronics) [Internet]. Wikipedia, The Free Encyclopedia; 4/1/2010. Available from: http://en.wikipedia.org/wiki/Adder_%28electronics%29.

[] Wikipedia contributors, D-Wave (electronics) [Internet]. Wikipedia, The Free Encyclopedia; 10/22/2014. Available from: http://en.wikipedia.org/wiki/D-Wave_Systems.

[] Wikipedia contributors, Quantum annealing [Internet]. Wikipedia, The Free Encyclopedia; 02/14/2015; Available from: http://en.wikipedia.org/wiki/Quantum_annealing



QC References

Picture and Graphic Credits:

Bloch_Sphere: http://comp.uark.edu/~jgeabana/blochapps/sphere2.jpg

Hadamard Transformation: derived from Brown, Figure 4.4, page 131.

Hadamard Matrix: derived from "Automatic Quantum Computer Programming", Lee Spector, 2007, Sprinter, ISBN: 978-0-387-36496-4, Sample pages: http://www.springer.com/cda/content/document/cda_downloaddocument/9780387364964-c2.pdf?SGWID=0-0-45-346665-p173670367 fulltextQCsim.pdf.

Digital Half Adder: http://en.wikipedia.org/wiki/Adder_%28electronics%29

Analog Computer Circuit: http://www.play-hookey.com/analog/analog_addition.html.

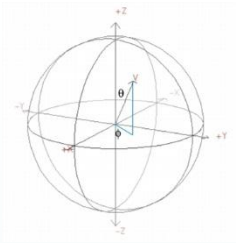
Quantum Hardware, NOT Gate (Pauli-x): derived from: Milburn, Figure 5.6, page 138.

H Gate Graphic: derived from Milburn, Figure 5.8, page 144.

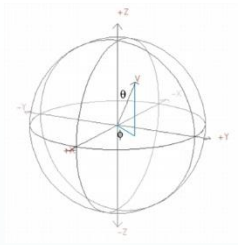
C-NOT Gate Graphic: derived from Yanofsky, Figure (6.6), page 172.

Deutsch's Algorithm Graphic: derived from Brown, Figure D-2, page 351.

Quantum Circuit Picture: http://almaak.usc.edu/~tbrun/Course/lecture05.pdf



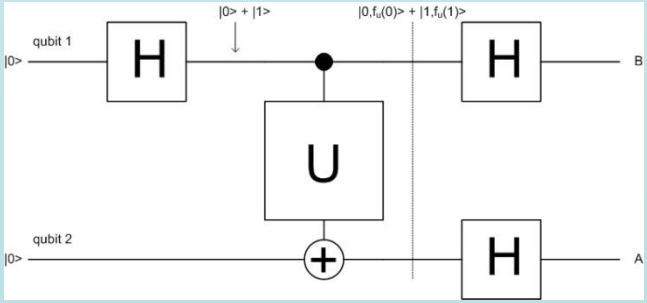
Slides Omitted for 5446



Quack! & Deutsch Algorithm

- There are 4 cases with 2 qubits: |10>, |01>, |00>, |11>.
- Each of these was run with Dcase1.m .. Dcase4.m.
- Results:

	qbits <>		**qbits <>**		**qbits =**		**qbits =**	
	Dcase1.m		**Dcase2.m**		**Dcase3.m**		**Dcase4.m**	
raw:								
bit 2	-1	1	-1	1	-1	1	-1	1
bit 1	-1	1	-1	1	1	-1	1	-1
formatted:		inconclusive		inconclusive		inconclusive		inconclusive
bit 2	|1>	|0>	|1>	|0>	|1>	|0>	|1>	|0>
bit 1	|1>	|0>	|1>	|0>	|0>	|1>	|0>	|1>



A: Measure first; if zero then inconclusive else measure B.

B: Measure second; if 0 then f values are the equal; if 1 then f values are not equal.

Note: We never get to see the actual f(qubit) results!